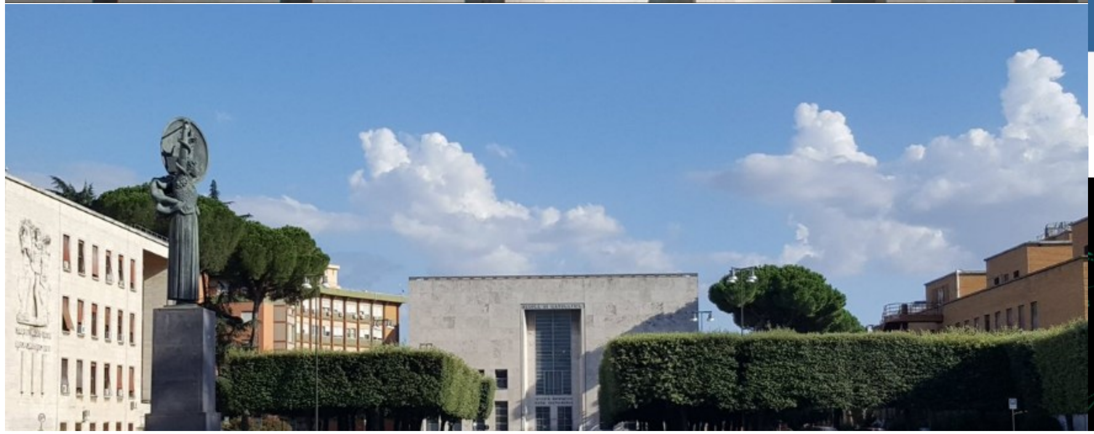
Complex photonics

Claudio Conti

NLP2019

Sapienza and ISC

University Sapienza
In Rome
(funded in 1303 AD)



erc

John Templeton Foundation

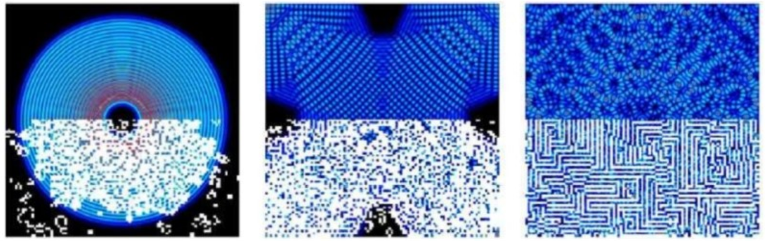
Sapienza team

Laura Pilozzi

Giulia Marcucci

Davide Pierangeli

Silvia Gentilini



www.newcomplexlight.org

Synopsis

01

Complex photonic devices

- Transmission matrix
  - Nonlinear transmission matrix
    - Applications (all-optical switching and bio)

02

Complex nonlinear dynamics

- Classical and quantum solitons
- Extreme waves

03

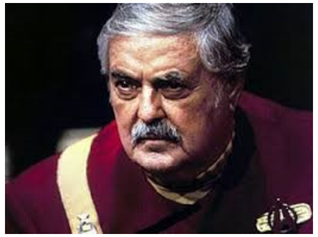
Numerical methods

- Beam Propagation Methods
- FCOMB solitons

A crazy idea for photonics - and engineering - in the new era of machine learning

OLD SCHOOL:

given an application, design and fabricate a device



NEW SCHOOL:

given a device, find a way to use it for your application

(…. not very new indeed

…. but we have new tools …

and we need a very complex device)

Outline

- Nonlinear complex systems by the transmission matrix
  - Green’s function
  - Propagator
  - Nonlinear perturbation to the propagator
  - Applications

- Classical and quantum optical solitons
  - The nonlinear Schroedinger equation
  - Numerical methods
  - Transition to dynamical complexity

Structural Vs Dynamical complexity

By morphology
- Random systems
- Complex arrays of waveguides
- Coupled cavities
- Biological systems

By nonlinearity
- Highly nonlinear regimes
- Many solitons
- Shocks and rogues waves
- Multimodal dynamics
- Ultrafast dynamics and plasmonics

Random lasers
(highly nonlinear and disordered systems)

Perturbative Vs non-perturbative extremes

Structural complexity

- Nonlinearity is a perturbation to tune or probe the systems

Dynamical complexity

- Nonlinearity is the leading actor in a nonperturbative regime

Structural complexity

Information processing

Ising machines and optical neuromorphic computing

Cryptography

Classical and quantum

Biomedicine

Drug delivery

Cancer treatments

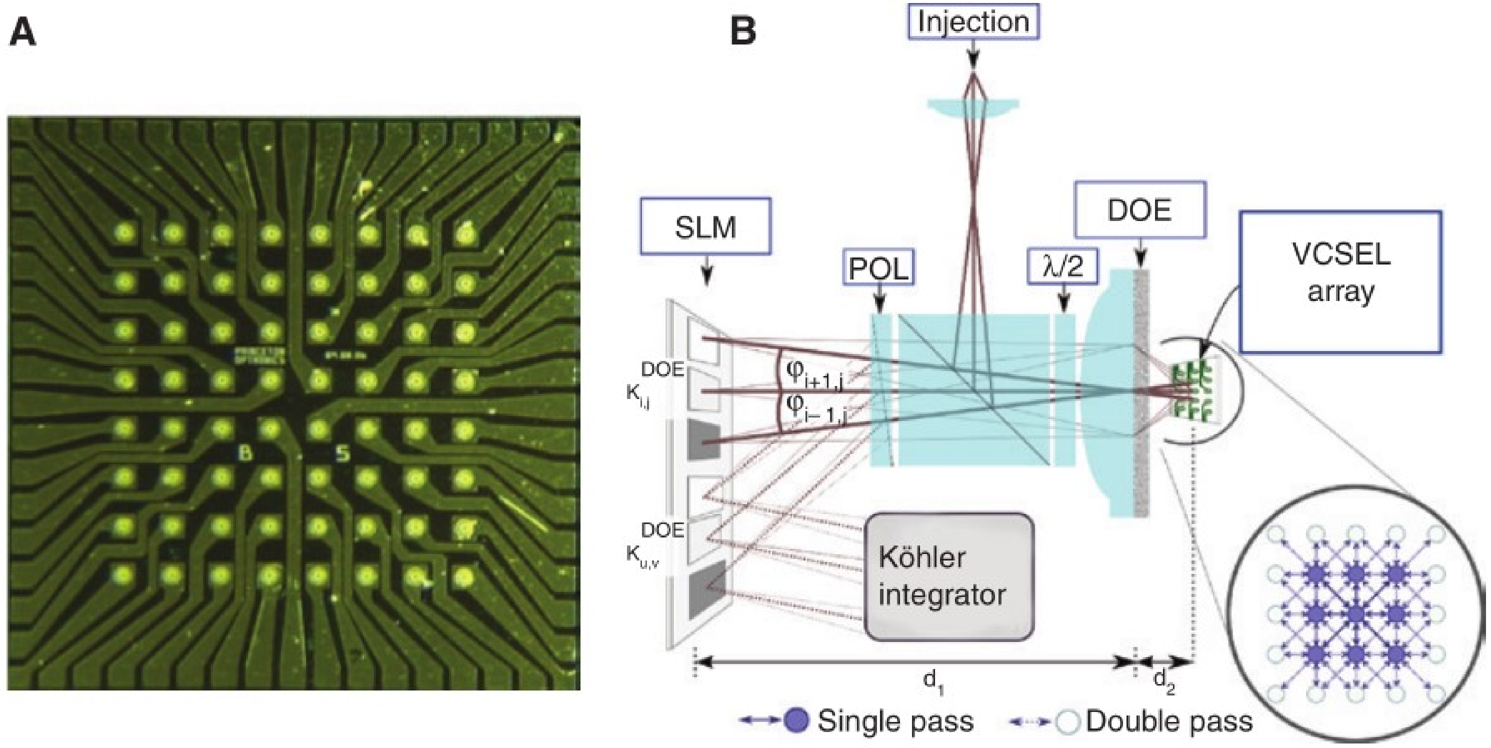
Microscopy

Sensors

Applications of structural complexity

Examples

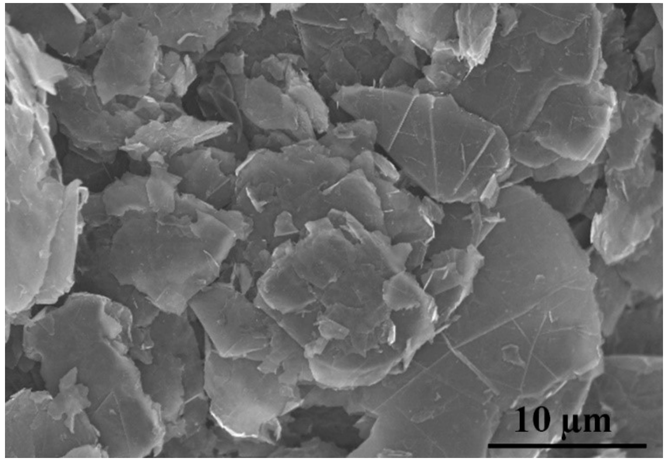
- Complex photonic circuits



Van der Sande, Nanophotonics 2017, 6, 531

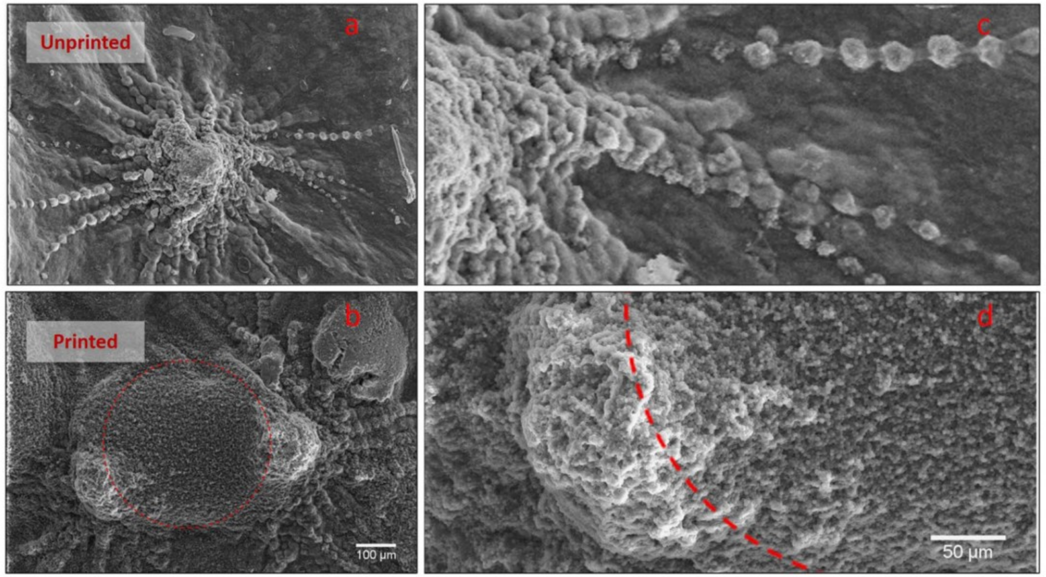
Examples

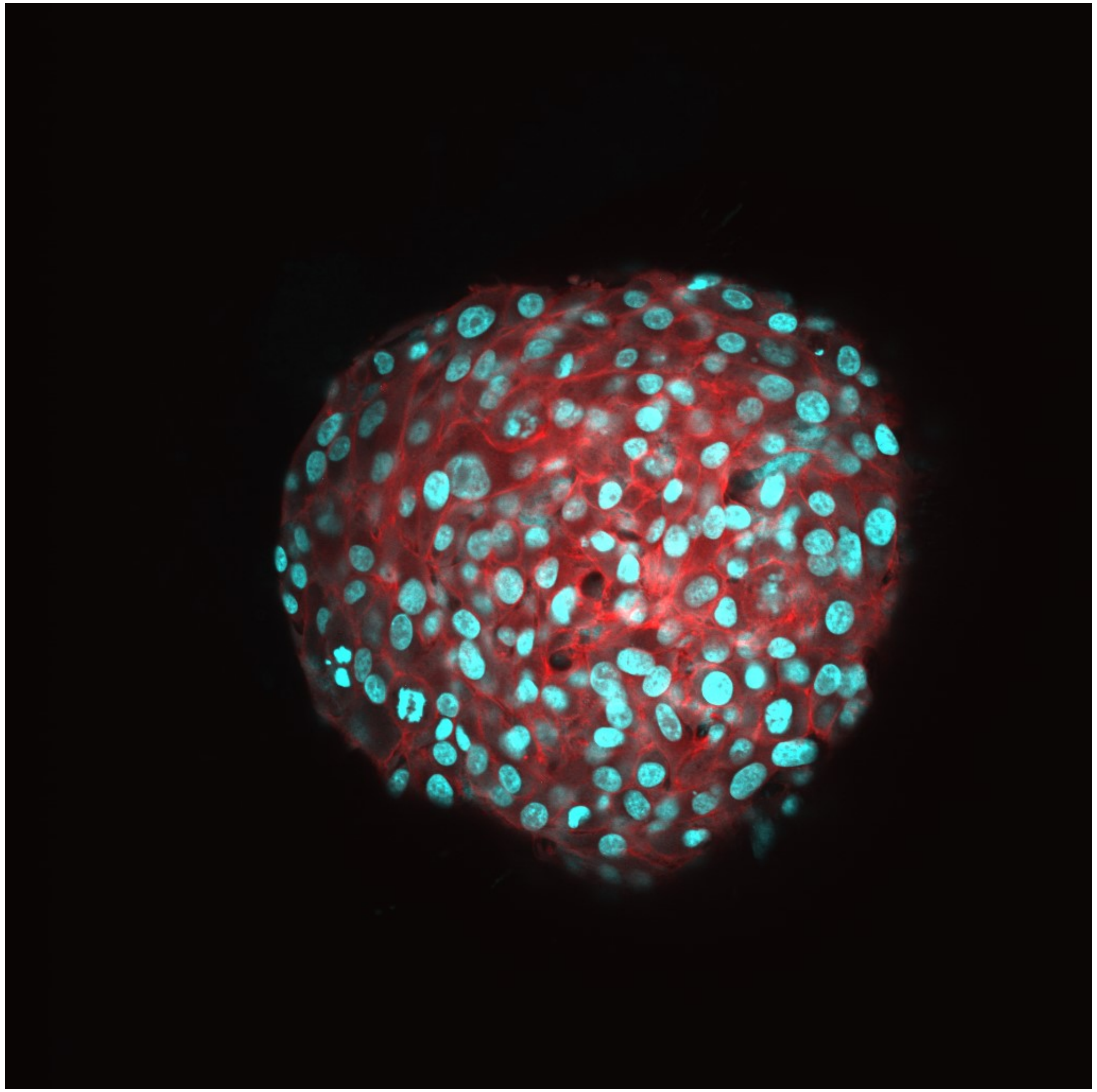
- Random systems



Examples

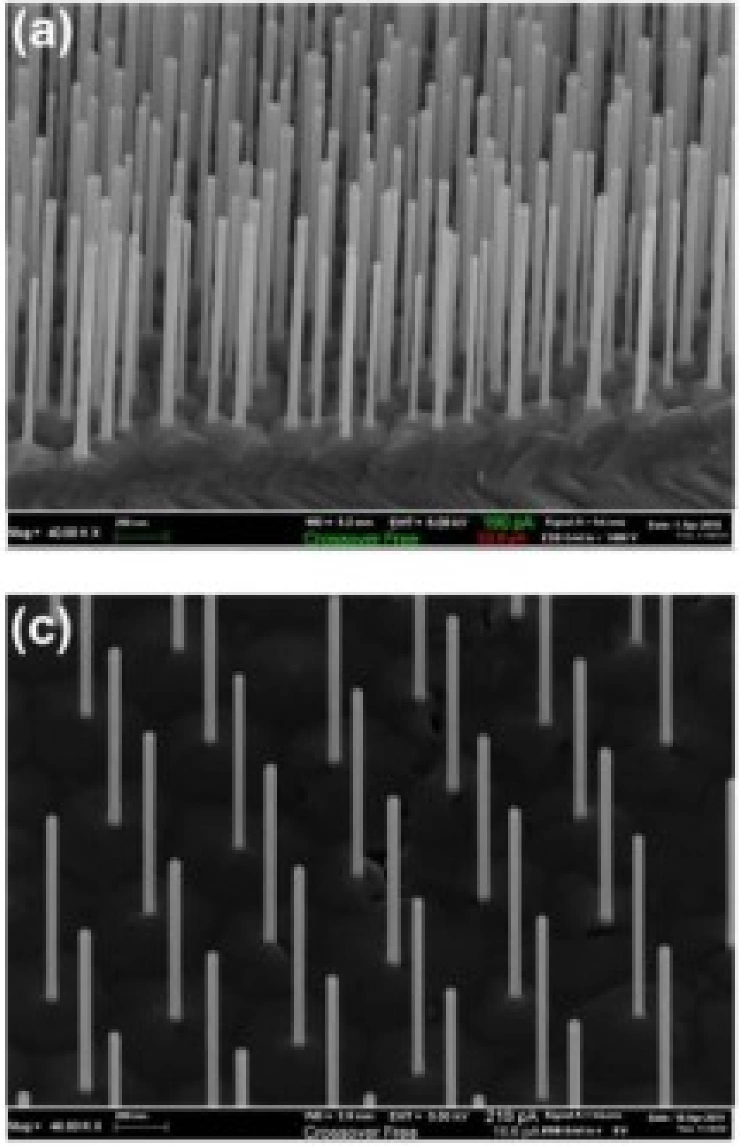
- Biological systems





Examples

- Metasurfaces



Light in complex media

Local outline

- Green’s function and Dirac notation
- Field propagator
- Transmission matrix
- Nonlinear perturbation to the transmission matrix

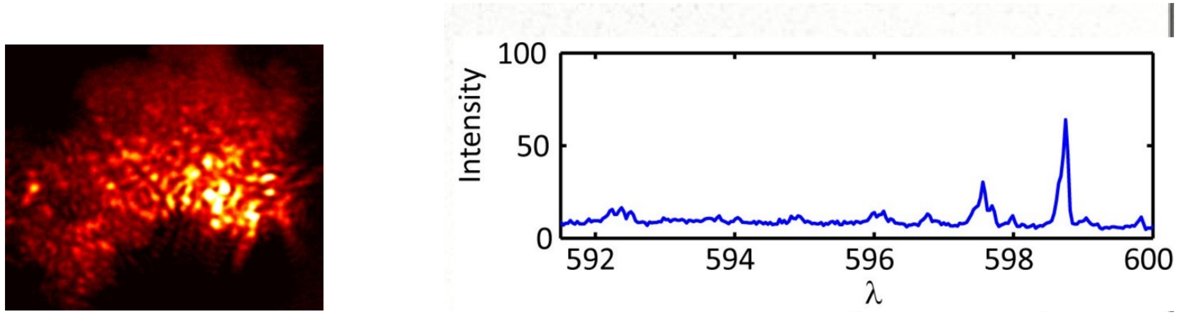
Green function and modes (scalar case)

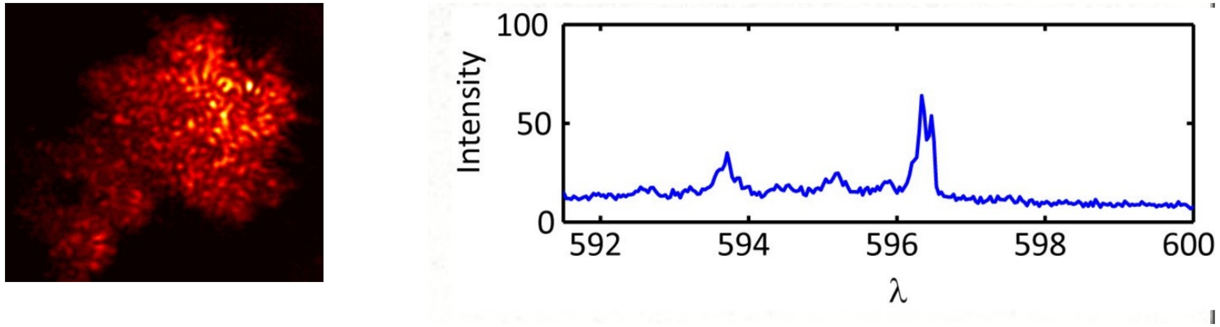
$$\nabla^2 E + k_0^2 \varepsilon_r(\mathbf{r}) E = 0$$

$$\nabla^2 G(\mathbf{r},\mathbf{r}') + k_0^2 \varepsilon_r(\mathbf{r}) G(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

$$-\nabla^2 \varphi_m = \varepsilon_r(r) \frac{\omega_n^2}{c^2} \varphi_m(r)$$

$$G(\mathbf{r},\mathbf{r}') = c^2 \sum_n \frac{\varphi_n(\mathbf{r}')^* \varphi_n(\mathbf{r})}{\omega^2 - \omega_n^2}$$





Leonetti, CC, Lopez, NPHOTON 5, 615 (2011)

Measure
the Green's function ?

The Green function is a complex quantity

$$G(\mathbf{r}, \mathbf{r}') = c^2 \sum_n \frac{\varphi_n(\mathbf{r}')^* \varphi_n(\mathbf{r})}{\omega^2 - \omega_n^2}$$

Green function (GF) at the resonance

$$\nabla^2 G + \frac{\omega^2}{c^2}\varepsilon_r(\mathbf{r})\,(1+2\imath\gamma)\,G = -\delta(\mathbf{r}-\mathbf{r}').$$

$$G(\mathbf{r},\mathbf{r}',\omega) = c^2\sum_n \frac{\varphi_n(\mathbf{r}')^*\varphi_n(\mathbf{r})}{\omega^2(1+\imath\gamma)^2-\omega_n^2}.$$

$$\frac{1}{\omega^2(1+\imath\gamma)^2-\omega_n^2} \cong PV\left(\frac{1}{\omega^2-\omega_n^2}\right) + \frac{\imath\pi}{2\omega_n}\delta(\omega-\omega_n)$$

$$\frac{1}{\omega^2-\omega_n^2} = PV\left(\frac{1}{\omega^2-\omega_n^2}\right) + \frac{\imath\pi}{2\omega}\delta(\omega-\omega_n)$$

NLP2019

Density of states and local density of states

$$\mathcal{N}(\omega) = \sum_n \delta(\omega - \omega_n)$$

DOS

$$\rho(\omega, \mathbf{r}) = \sum_n \delta(\omega - \omega_n)\varphi_n(\mathbf{r})^* \varphi_n(\mathbf{r})$$

LDOS

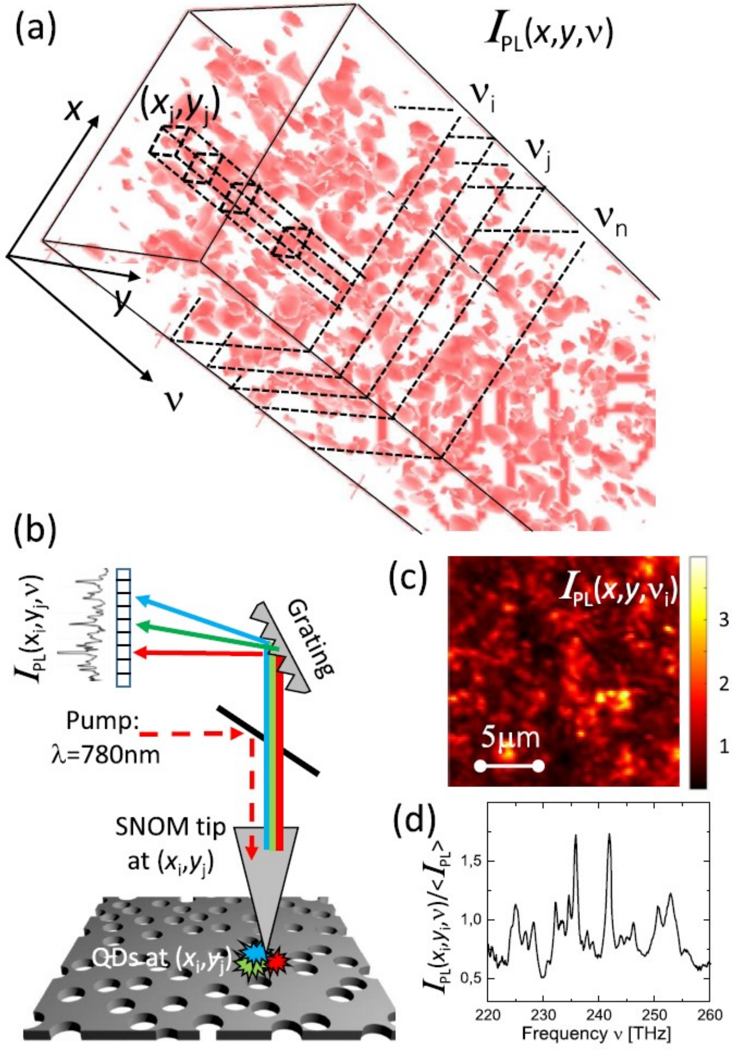
LDOS is the imaginary part of the GF

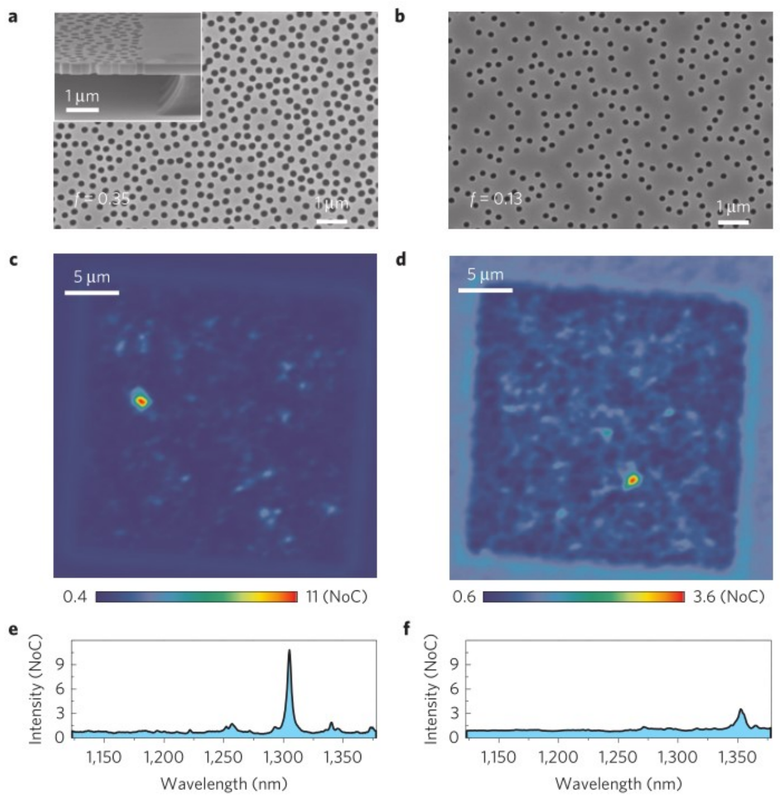
$$G(\mathbf{r},\mathbf{r}',\omega) = c^2 \sum_n \varphi_n^*(\mathbf{r}')\varphi_n(\mathbf{r}) \left[PV\left(\frac{1}{\omega_n^2-\omega^2}\right) + \imath \frac{\pi}{2\omega_n}\delta(\omega-\omega_n)\right]$$

$$\Im G(\mathbf{r},\mathbf{r}',\omega) = \frac{\pi c^2}{2\omega}\sum_n \varphi_n(\mathbf{r}')^*\varphi_n(\mathbf{r})\delta(\omega-\omega_n).$$

$$\rho(\mathbf{r},\omega) = \frac{2\omega}{\pi c^2}\Im\left[G(\mathbf{r},\mathbf{r},\omega)\right].$$

Local density of states (LDOS)





ARTICLES
PUBLISHED ONLINE: 18 MAY 2014 | DOI: 10.1038/NMAT3966
nature materials

Engineering of light confinement in strongly scattering disordered media

Francesco Riboli[1,2*†], Niccolò Caselli[1,2], Silvia Vignolini[1,2†], Francesca Intonti[1,2], Kevin Vynck[1†], Pierre Barthelemy[1†], Annamaria Gerardino[3], Laurent Balet[4], Lianhe H. Li[4], Andrea Fiore[4†], Massimo Gurioli[1,2] and Diederik S. Wiersma[1,2]

PRL 119, 043902 (2017) PHYSICAL REVIEW LETTERS week ending 28 JULY 2017

Tailoring Correlations of the Local Density of States in Disordered Photonic Materials

F. Riboli,[1,2,3,*] F. Uccheddu,[4] G. Monaco,[1] N. Caselli,[3,5,6] F. Intonti,[3,5] M. Gurioli,[3,5] and S. E. Skipetrov[7,8,†]

Canonical notation for the Green's function

$$\nabla^2 G(\mathbf{r},\mathbf{r}') + k_0^2 \varepsilon_r(\mathbf{r}) G(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

$$[z - L(\mathbf{r})]G(\mathbf{r},\mathbf{r}';z) = \delta(\mathbf{r}-\mathbf{r}')$$

$$L(\mathbf{r})\phi_n(\mathbf{r}) = \lambda_n \phi_n(\mathbf{r}) \qquad \sum_n \phi_n(\boldsymbol{r})\phi_n^*(\boldsymbol{r}') = \delta(\boldsymbol{r}-\boldsymbol{r}') \; .$$

Dirac notation for classical fields

$$\phi_n(\mathbf{r}) = \langle \mathbf{r} | \phi_n \rangle$$

$$\phi_n(\mathbf{r})^* = \langle \phi_n | \mathbf{r} \rangle$$

$$\delta(\mathbf{r} - \mathbf{r}')L(\mathbf{r}) = \langle \mathbf{r} | L | \mathbf{r}' \rangle$$

$$G(\mathbf{r}, \mathbf{r}'; z) = \langle \mathbf{r} | G(z) | \mathbf{r}' \rangle$$

$$\langle \mathbf{r} | \mathbf{r}' \rangle = \delta(\mathbf{r} - \mathbf{r}')$$

$$\int \mathrm{d}\mathbf{r} | \mathbf{r} \rangle \langle \mathbf{r} | = 1$$

See Economou, *Green's function in Quantum Physics*

Green function in the Dirac notation

$$(z - L)\,G(z) = 1$$

$$[z - L(\mathbf{r})]G(\mathbf{r}, \mathbf{r}'; z) = \delta(\mathbf{r} - \mathbf{r}')$$

$$L|\phi_n\rangle = \lambda_n|\phi_n\rangle$$

$$G(z) = \frac{1}{z - L} = \sum_n \frac{|\phi_n\rangle\langle\phi_n|}{z - \lambda_n}$$

Green's function, vectorial case

$$-\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) + \frac{\omega^2}{c^2}\varepsilon_r(\mathbf{r})\mathbf{E}(\mathbf{r}) = \imath\mu_0\omega\mathbf{J}(\mathbf{r})$$

$$\mathbf{E}(\mathbf{r}) = \imath\mu_0\omega \int \mathrm{d}\mathbf{r}'\mathbf{G}(\mathbf{r},\mathbf{r}';\omega)\cdot\mathbf{J}(\mathbf{r}')$$

$$-\nabla \times \nabla \times \mathbf{G}(\mathbf{r},\mathbf{r}';\omega) + \frac{\omega^2}{c^2}\varepsilon_r(\mathbf{r})\mathbf{G}(\mathbf{r},\mathbf{r}';\omega) = \delta(\mathbf{r}-\mathbf{r}')\mathbf{I}$$

Non-canonical modal set

The vectorial equation for the EM field is

$$\nabla \times \nabla \times \mathbf{E} - \varepsilon_r(\mathbf{r})\frac{\omega^2}{c^2}\mathbf{E} = 0$$

we define the modes

$$\nabla \times \nabla \times \mathbf{e}_n - \varepsilon_r(\mathbf{r})\frac{\omega_n^2}{c^2}\mathbf{e}_n = 0$$

which obey

$$\int_V \varepsilon_r(\mathbf{r})\mathbf{e}_m \cdot \mathbf{e}_n^* \, dV = \delta_{mn}.$$

the *transversality condition*

$$\nabla \cdot (\varepsilon_r(\mathbf{r})\mathbf{e}_n(\mathbf{r})) = 0$$

For these modes the definition of the DOS is

$$\mathcal{N}(\omega) = \sum_n \delta(\omega - \omega_n),$$

but the LDOS needs to accunt for the vectorial nature of the $\mathbf{e}_m$, and we have

$$\rho(\mathbf{r}, \omega) = \sum_n |\mathbf{e}_n(\mathbf{r})|^2 \delta(\omega - \omega_n)$$

The modes $\mathbf{e}_n$ are not orthogonal in the usual sense - note the quantity $\varepsilon_r(\mathbf{r})$ in and the operator $\nabla \times \nabla \times$ is not Hermitian, hence we cannot map directly to the general formalism for the Green function

NLP2019

Canonical modes for Maxwell equations

$$\phi_n(\mathbf{r}) = \sqrt{\varepsilon_r(\mathbf{r})}\mathbf{e}_n(\mathbf{r}).$$

$$\mathbf{L}(\mathbf{r})\phi_m = \frac{1}{\sqrt{\varepsilon_r(\mathbf{r})}}\nabla\times\left[\nabla\times\frac{\phi_m}{\sqrt{\varepsilon_r(\mathbf{r})}}\right] = \frac{\omega_m^2}{c^2}\phi_m(\mathbf{r}).$$

$$\int \phi_m^*(\mathbf{r})\cdot\phi_n(\mathbf{r})\,\mathrm{d}\mathbf{r} = \delta_{mn}$$

PHYSICAL REVIEW A VOLUME 43, NUMBER 1 1 JANUARY 1991

Quantum optics of dielectric media

Roy J. Glauber and M. Lewenstein*
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 20 February 1990)

Properties of the canonical set

$$\nabla\left[\cdot\sqrt{\varepsilon_r(\mathbf{r})}\phi_m\right]=0.$$

$$\sum_m \phi_m(\mathbf{r})\cdot\phi_m^*(\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')$$

Dirac notation in the vectorial case

$$[z - \mathbf{L}(\mathbf{r})]\,\mathbf{G}_L(\mathbf{r},\mathbf{r}') = \left\{ z\mathbf{G}_L(\mathbf{r},\mathbf{r}') - \frac{1}{\sqrt{\varepsilon_r(\mathbf{r})}} \nabla \times \left[\nabla \times \frac{\mathbf{G}_L}{\sqrt{\varepsilon_r(\mathbf{r})}} \right] \right\} = \mathbf{I}\delta(\mathbf{r}-\mathbf{r}')$$

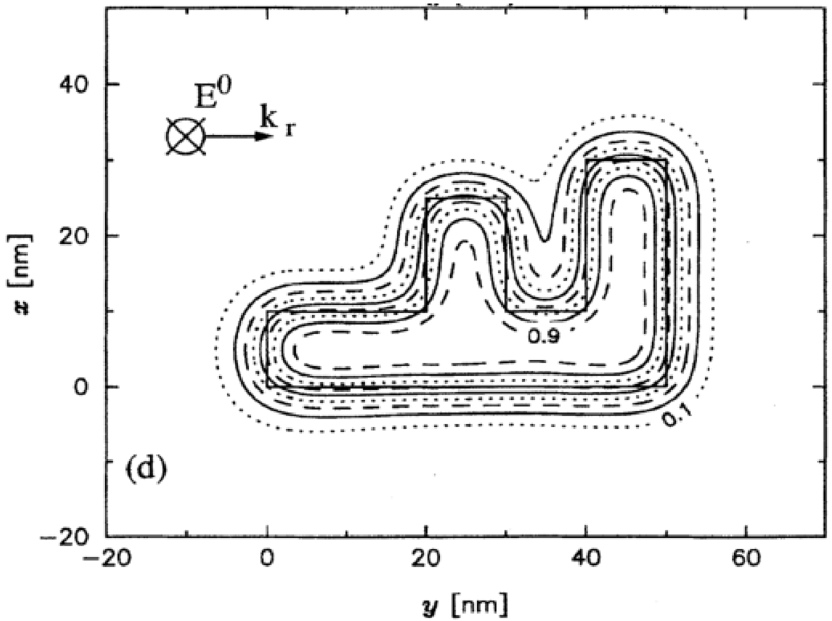
$$\mathbf{G}(z) = \frac{1}{z - \mathbf{L}} \sum_m |\boldsymbol{\phi}_m\rangle\langle\boldsymbol{\phi}_m| = \sum_m \frac{|\boldsymbol{\phi}_m\rangle\langle\boldsymbol{\phi}_m|}{z - \lambda_m}$$

$$\langle\mathbf{r}|\boldsymbol{\phi}_m\rangle\langle\boldsymbol{\phi}_m|\mathbf{r}'\rangle = \boldsymbol{\phi}_m^*(\mathbf{r}') \otimes \boldsymbol{\phi}_m(\mathbf{r})$$

$$\mathbf{G}_L(\mathbf{r},\mathbf{r}',\omega) = c^2 \sum_m \frac{\boldsymbol{\phi}_m^*(\mathbf{r}') \otimes \boldsymbol{\phi}_m(\mathbf{r})}{\omega^2 - \omega_m^2}$$

Field propagator

Field propagator



$$\varepsilon_r(\mathbf{r}) = \varepsilon_b(\mathbf{r}) + \varepsilon_s(\mathbf{r})$$

$$-\nabla \times \nabla \times \mathbf{E} + k_0^2 \left[\varepsilon_b(\mathbf{r}) + \varepsilon_s(\mathbf{r})\right] \mathbf{E} = 0.$$

$$\langle \mathbf{r}|\mathbf{E}\rangle = \mathbf{E}(\mathbf{r})$$
$$\langle \mathbf{r}|\mathbf{E}_0\rangle = \mathbf{E}_0(\mathbf{r})$$

VOLUME 74, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JANUARY 1995

Generalized Field Propagator for Electromagnetic Scattering and Light Confinement

Olivier J. F. Martin*
IBM Research Division, Zurich Research Laboratory, 8803 Rueschlikon, Switzerland

Christian Girard
Laboratoire de Physique Moléculaire UA CNRS 772, Université de Franche Comté, 2530 Besançon, France

Alain Dereux
Institute for Studies in Interface Sciences, Facultés Universitaires N.–D. de la Paix, 5000 Namur, Belgium
(Received 9 August 1994)

Field propagator in Dirac notation

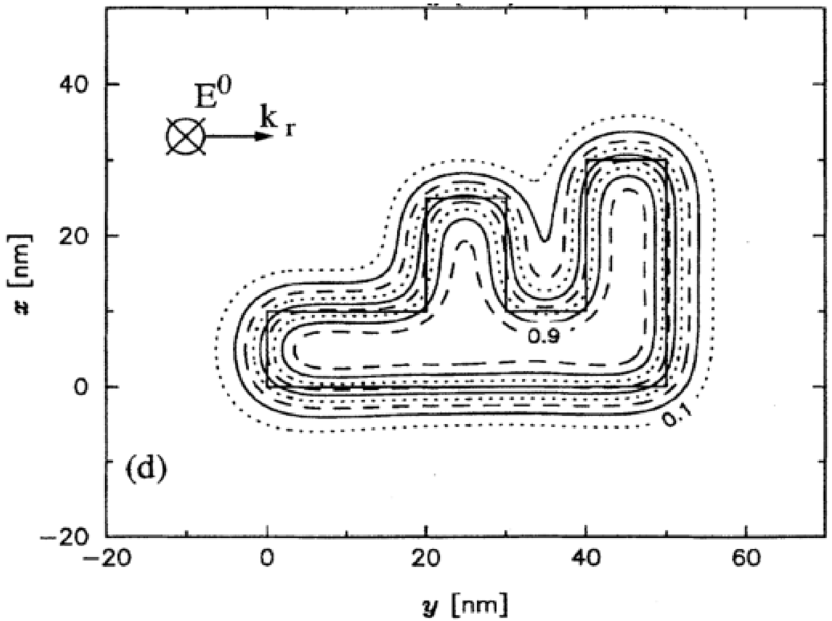
$$-\nabla \times \nabla \times \mathbf{E} + k_0^2 \varepsilon_r(\mathbf{r})\mathbf{E} = 0.$$

$$\boldsymbol{\mathcal{D}}(\mathbf{r}) = -\nabla \times \nabla \times,$$

$$(\boldsymbol{\mathcal{D}} + \mathbf{e})\,|\mathbf{E}\rangle = 0$$

$$\langle \mathbf{r}|\mathbf{E}\rangle = \mathbf{E}(\mathbf{r}) \qquad \langle \mathbf{r}|\mathbf{e}|\mathbf{r}'\rangle = k_0^2 \varepsilon_r(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}'),$$

Input field and total field



$$\mathbf{e} = \mathbf{e}_b + \mathbf{e}_s$$

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b)\,|\mathbf{E}_0\rangle = 0,$$

Input beam (plane wave, Gaussian beam, etc…)

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}\rangle = 0$$

Total field

The Green's function

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,\mathbf{G} = \mathbf{1}.$$

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}\rangle = (\boldsymbol{\mathcal{D}} + \mathbf{e}_b)\,|\mathbf{E}_0\rangle = 0,$$

From the Green's function to the propagator

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}\rangle = (\boldsymbol{\mathcal{D}} + \mathbf{e}_b)\,|\mathbf{E}_0\rangle = 0,$$

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}\rangle = (\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}_0\rangle - \mathbf{e}_s|\mathbf{E}_0\rangle,$$

$$(\boldsymbol{\mathcal{D}} + \mathbf{e}_b + \mathbf{e}_s)\,\mathbf{G} = \mathbf{1}.$$

$$|\mathbf{E}\rangle = |\mathbf{E_0}\rangle - \mathbf{G}\mathbf{e}_s|\mathbf{E}_0\rangle.$$

$$|\mathbf{E}\rangle = \mathbf{K}|\mathbf{E}_0\rangle \qquad \mathbf{K} = \mathbf{1} - \mathbf{G}\mathbf{e_s}.$$

$$\langle\mathbf{r}|\mathbf{K}|\mathbf{r}'\rangle = \mathbf{1}\delta(\mathbf{r} - \mathbf{r}') - k_0^2\varepsilon_r(\mathbf{r}')\langle\mathbf{r}|\mathbf{G}|\mathbf{r}'\rangle.$$

Transmission matrix

Transmission matrix definition

$$|\mathbf{E}\rangle = \mathbf{K}|\mathbf{E_0}\rangle$$

$$|\mathbf{E}\rangle = \sum_n c_{out,n}|n\rangle$$

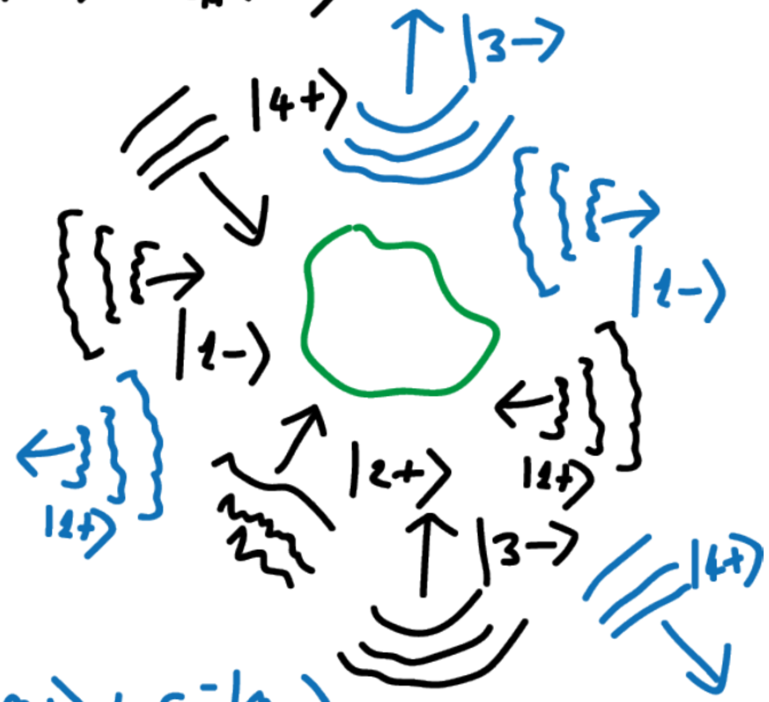
«CHANNELS»

$$|\mathbf{E_0}\rangle = \sum_n c_{in,n}|n\rangle$$

$$E_m^{out} = \sum_n k_{mn} E_n^{in}$$

$$|E_0\rangle = c_m^+ |m+\rangle + c_m^- |m-\rangle$$

«CHANNELS»



$$|E\rangle = c_m^+ |m+\rangle + c_m^- |m-\rangle$$

The transfer matrix is unitary

$$E_m^{out} = \sum_n k_{mn} E_n^{\text{in}}$$

$$\sum_m \left| E_m^{\text{in}} \right|^2 = \sum_m \left| E_m^{out} \right|^2$$

$$k_{mn} = k_{nm}^*$$

$$K^\dagger = K$$

Measurement of the transmission matrix

$$I_m = |s_m + \sum_n e^{\imath\alpha} k_{mn} E_n^{\text{in}}|^2 = |s_m|^2 + |\sum_n e^{\imath\alpha} E_n^{\text{in}}|^2 + 2\Re\left(e^{\imath\alpha} s_m^* \sum_n k_{mn} E_n^{\text{in}}\right)$$

In the four phases method, one makes four measurements with $\alpha = 0$, $\alpha = \pi/2$, $\alpha = \pi$ and $\alpha = 3\pi/2$. Correspondingly one has

$$\begin{aligned}
I_m^0 &= |s_m|^2 + |\sum_n k_{mn} E_n|^2 + 2\Re\left(s_m^* \sum_n k_{mn} E_n^{\text{in}}\right)\\
I_m^{\pi/2} &= |s_m|^2 + |\sum_n k_{mn} E_n|^2 - 2\Im\left(s_m^* \sum_n k_{mn} E_n^{\text{in}}\right)\\
I_m^{\pi} &= |s_m|^2 + |\sum_n k_{mn} E_n|^2 - 2\Re\left(s_m^* \sum_n k_{mn} E_n^{\text{in}}\right)\\
I_m^{3\pi/2} &= |s_m|^2 + |\sum_n k_{mn} E_n|^2 + 2\Im\left(s_m^* \sum_n k_{mn} E_n^{\text{in}}\right)
\end{aligned}$$

Combining the previous equation, we have

$$\frac{1}{4}\left(I_m^0 - I_m^\pi\right) + \frac{\imath}{4}\left(I_m^{3\pi/2} - I_m^{\pi/2}\right) = s_m^* \sum_n k_m E_n^{\text{in}}$$

If one inject only the input mode n such that E_n^{in} is one only for a particular n, one has

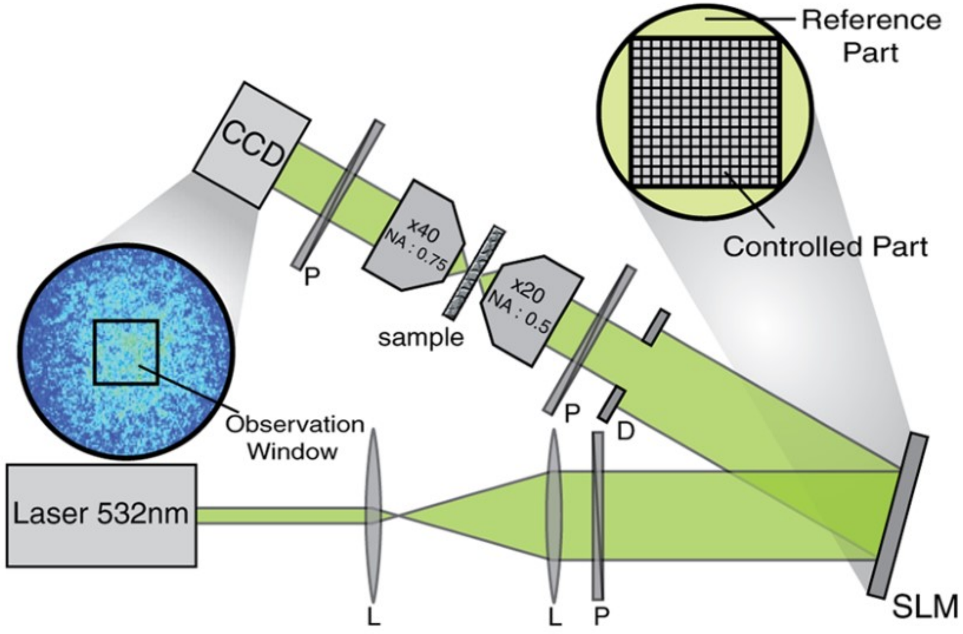
$$\frac{1}{4}\left(I_m^0 - I_m^\pi\right) + \frac{\imath}{4}\left(I_m^{3\pi/2} - I_m^{\pi/2}\right) = s_m^* k_{mn}$$

The quantity s_m is in general different for all the modes, but in practical applications it is just a scaling factor in the matrix elements k_{mn} that is nearly the same for all modes. A proper measurement would require a complex interferometrix setup, however a simple and feasible approach is observing that the transmission matrix as diagonal elements of the order of unity, hence one can estimate its average by the mean of the value $s_m^* k_{mn}$ when varying n:

$$\langle s_m^* \rangle = \frac{1}{N} \sum s_m k_{mn}$$

and approximate the transmission matrix as

$$k_{mn} \cong \frac{s_m^*}{\langle s_m^* \rangle} k_{mn}$$



Selected for a Viewpoint in *Physics*
RL **104,** 100601 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

Measuring the Transmission Matrix in Optics: An Approach to the Study and Control of Light Propagation in Disordered Media

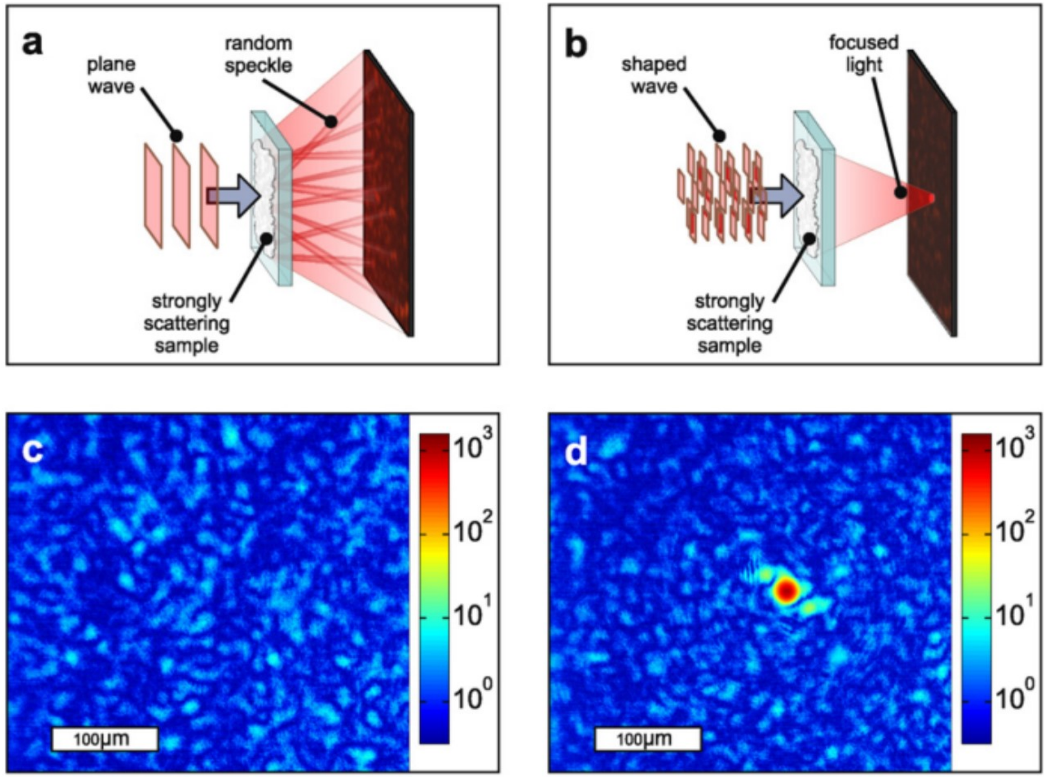
S. M. Popoff, G. Lerosey, R. Carminati, M. Fink, A. C. Boccara, and S. Gigan
Institut Langevin, ESPCI ParisTech, CNRS UMR 7587, ESPCI, 10 rue Vauquelin, 75005 Paris, France

(Received 27 October 2009; revised manuscript received 11 January 2010; published 8 March 2010)

Focusing light in complex media

Application of the transfer matrix

The Vellekoop and Mosk experiment



August 15, 2007 / Vol. 32, No. 16 / OPTICS LETTERS 2309

Focusing coherent light through opaque strongly scattering media

I. M. Vellekoop* and A. P. Mosk

Guidestar assisted wavefront shaping

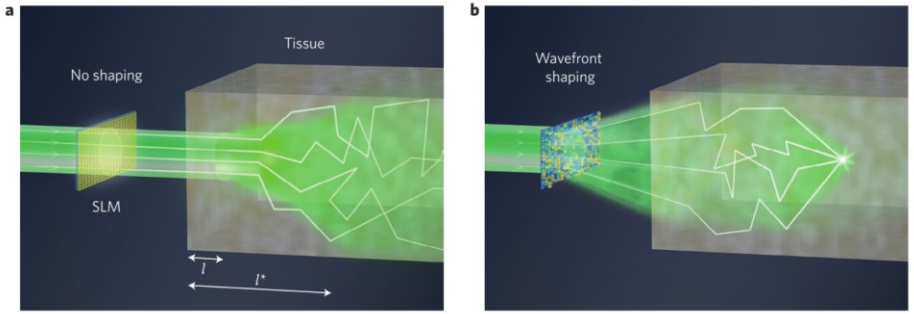


Figure 1 | Principle of wavefront shaping. a, An unmodified coherent beam of light travels one mean free path (l) with minimal scattering into tissue. A fraction of beam directionality is preserved up to the transport mean free path length, l^*. **b**, By wavefront-shaping the incident field with an SLM, it is possible to focus within tissue beyond l^*.

nature photonics | REVIEW ARTICLE
PUBLISHED ONLINE: 27 AUGUST 2015 | DOI: 10.1038/NPHOTON.2015.140

Guidestar-assisted wavefront-shaping methods for focusing light into biological tissue

Roarke Horstmeyer*, Haowen Ruan and Changhuei Yang

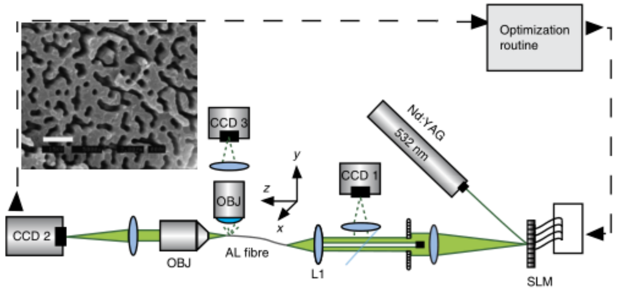
nature COMMUNICATIONS

ARTICLE

Received 17 Jan 2014 | Accepted 27 Jun 2014 | Published 29 Jul 2014 DOI: 10.1038/ncomms5534

Light focusing in the Anderson regime

Marco Leonetti[1,2], Salman Karbasi[3], Arash Mafi[3] & Claudio Conti[4]



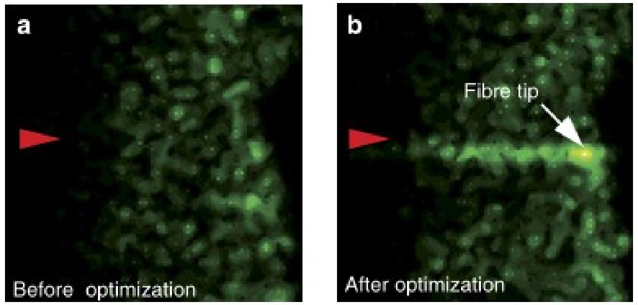


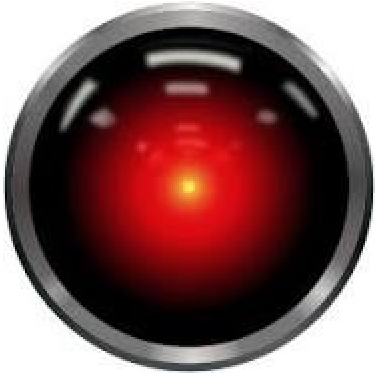
Figure 6 | Localized mode and adaptive focus. Light scattered from the side of the fibre in correspondence of the exit tip (**a**) before the optimization process and (**b**) after the optimization process. The side of the panels is 160 μm.

Focusing in (random) waveguides

Focusing in a single point is a simple form of optical machine learning

What is an «ARTIFICIAL NEURAL NETWORK»?

Is it a magic mathematical object that displays intelligence ?



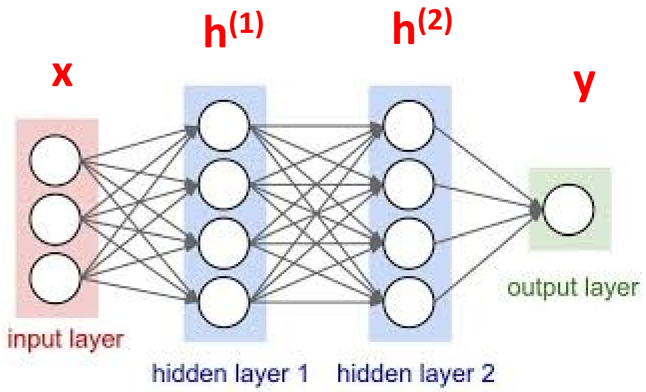
- May be IT IS !

- But - perhaps - is just a very useful «UNIVERSAL» fitting function !

Artificial Neural Network = Universal Interpolator

$$\mathbf{y} = f(\mathbf{x};\mathbf{a},\mathbf{b})$$

A universal fitting function that takes a N-dimensional input x and has output y that can be tuned by acting on the parameters



$$h_m = \sum_n a_{mn} x_n + b_m$$

$$y = |h_m|^2 = |\sum_n a_{mn} x_n + b_m|^2$$

Assume that you want to focalize light

- Solution 1: You take a lens

- Solution 2: you take any kind of transparent «complex» device
  - complex photonic sample(=coupled waveguides, fiber, random medium, etc…)
  - find the way to have fitting parameters (=SLM, nonlinearity, electrooptics, …)
  - and train it (…many strategies …)

- A device that focuses light is

an optical function that maps a plane wave in a single spot

- We can use a universal interpolator to realize it

Focusing a plane wave as a neural network

$$E_n^{\mathrm{in}} = A_n e^{i\alpha_n}$$

Plane wave input

$$A_n = A$$

$$\alpha_n = 0$$

$$I = |A|^2 = 1/N,$$

$$x_n = A$$

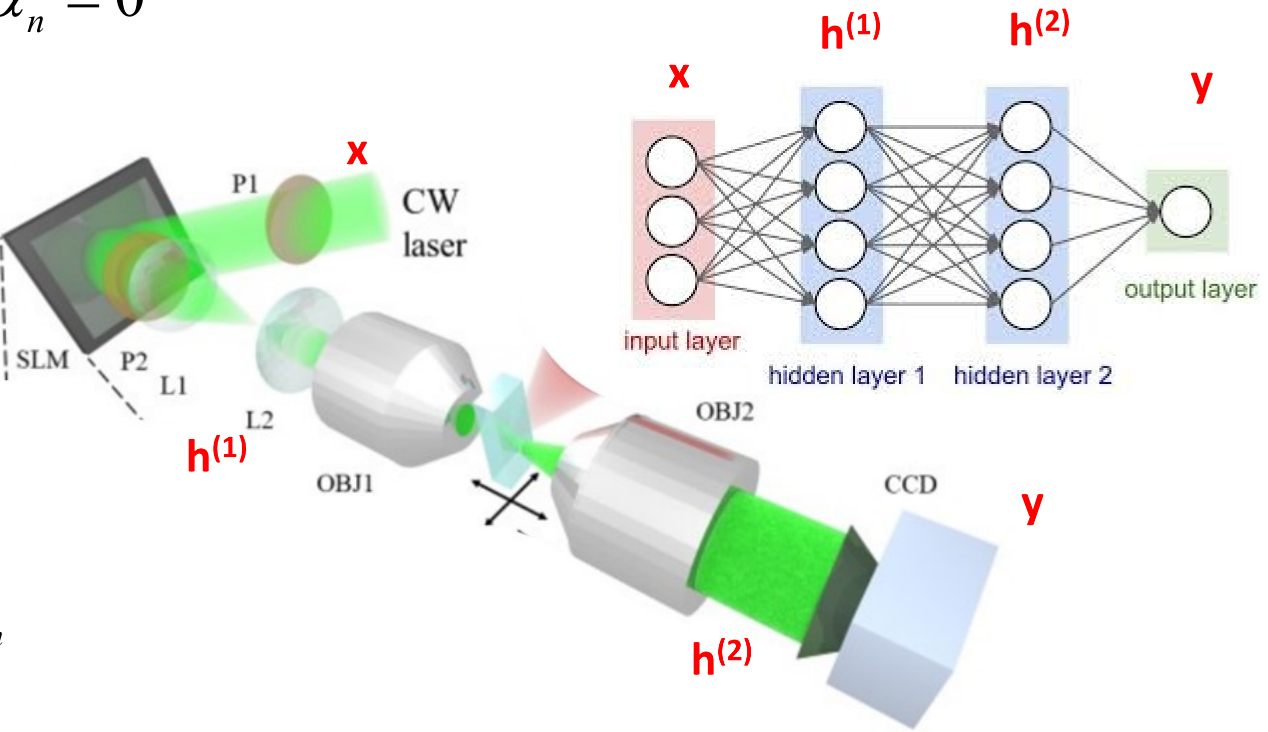
$$E_n^{SLM} = A e^{i\phi_n}$$

$$h_n^{(1)} = x_n e^{i\phi_n}$$

$$E_m^{out} = A \sum_n k_{mn}\ e^{i\phi_n}$$

$$h_m^{(2)} = \sum_n k_{mn} x_n e^{i\phi_n}$$

$$y = \left| A \sum_n k_{mn}\ e^{i\phi_n} \right|^2$$



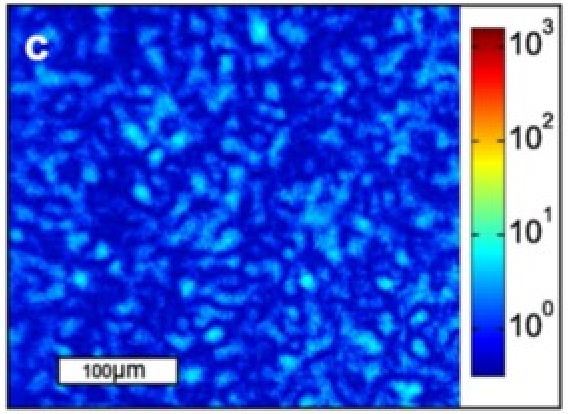
No training in the case of a random medium

$$E_n^{SLM} = Ae^{i\phi_n} = \sqrt{\frac{1}{N}}e^{i\phi_n}$$

$$|E_m^{\text{out}}|^2 = \frac{1}{N}|\sum_{n=1}^{N} k_{mn}e^{\imath\phi_n}|^2$$

No training

$$\phi_n = 0$$

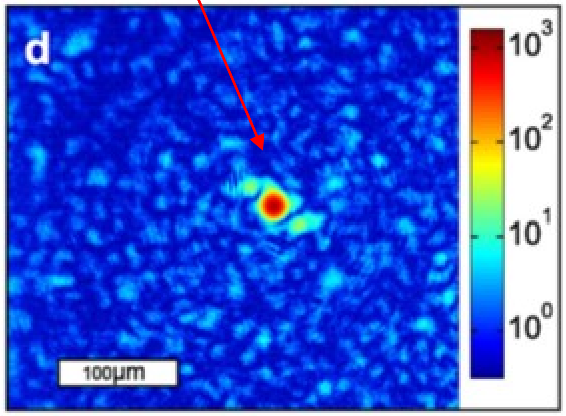


$$\langle I_0\rangle = \langle\frac{1}{N}|\sum_{n=1}^{N} k_{mn}|^2\rangle = \frac{1}{N}\sum_{n}\langle|k_{mn}|^2\rangle = \langle|k_{mn}|^2\rangle$$

The background intensity provides information on the mean matrix element

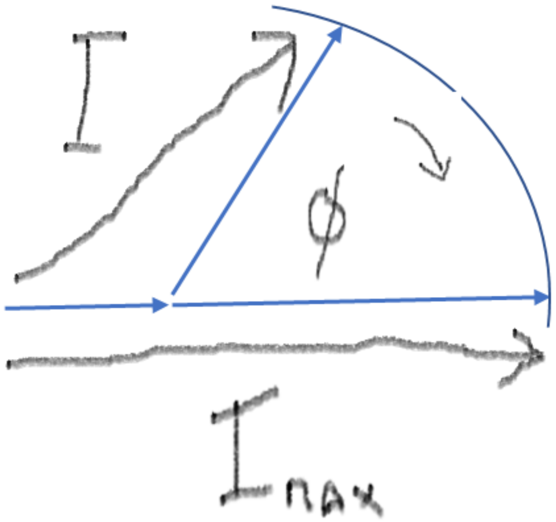
Single point focusing: «exact solution»

Mode m



$$|E_m^{\text{out}}|^2 = \frac{1}{N} \left| \sum_{n=1}^{N} k_{mn} e^{\imath \phi_n} \right|^2$$

$$\phi_n = -\arg\left(k_{mn}\right)$$



$$I_{max} = \frac{1}{N} \left(\sum_n |k_{mn}| \right)^2 = \frac{1}{N} \sum_q |k_{mq}| \sum_n |k_{mn}|$$

The number of modes and the enhancement

$$I_{max} = \frac{1}{N}\left(\sum_n |k_{mn}|\right)^2 = \frac{1}{N}\sum_q |k_{mq}| \sum_n |k_{mn}|$$

$$I_{max} = \frac{1}{N}\sum_n |k_{mn}|^2 + \frac{1}{N}\sum_n \sum_{q\neq n} |k_{mn}||k_{mq}|$$

$$\langle I_{max}\rangle = \langle I_0\rangle + \frac{1}{N}\sum_n \sum_{q\neq n}\langle|k_{mn}|\rangle\langle|k_{mq}|\rangle$$

$$\langle I_0\rangle = \langle\frac{1}{N}|\sum_{n=1}^{N} k_{mn}|^2\rangle = \frac{1}{N}\sum_n \langle|k_{mn}|^2\rangle = \langle|k_{mn}|^2\rangle$$

$$\langle|k_{mn}|\rangle = \sqrt{\pi}2\sigma = \frac{\sqrt{\pi\langle I_0\rangle}}{2}$$

$$\boxed{\langle I_{max}\rangle = \langle I_0\rangle\left[\frac{\pi}{4}(N-1)+1\right]}$$

$$\boxed{\eta = \frac{\langle I_{max}\rangle}{\langle I_0\rangle} = \frac{\pi}{4}(N-1)+1 \simeq \frac{\pi}{4}N}$$

August 15, 2007 / Vol. 32, No. 16 / OPTICS LETTERS 2309

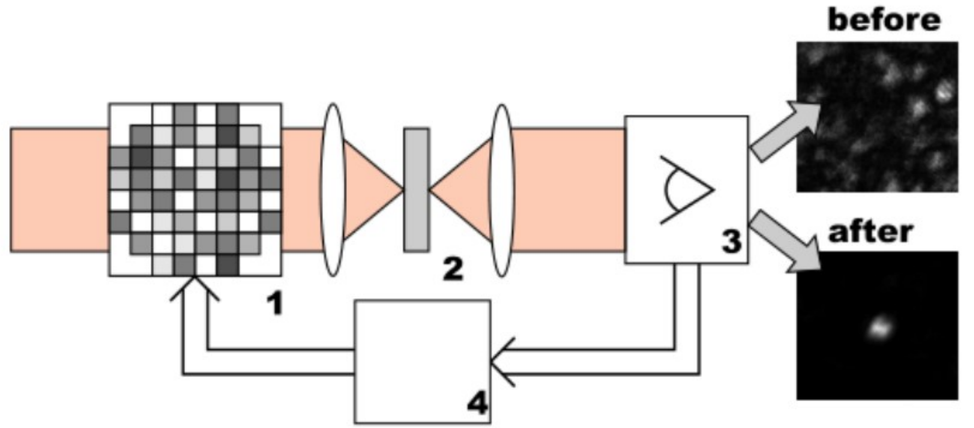
Focusing coherent light through opaque strongly scattering media

I. M. Vellekoop* and A. P. Mosk

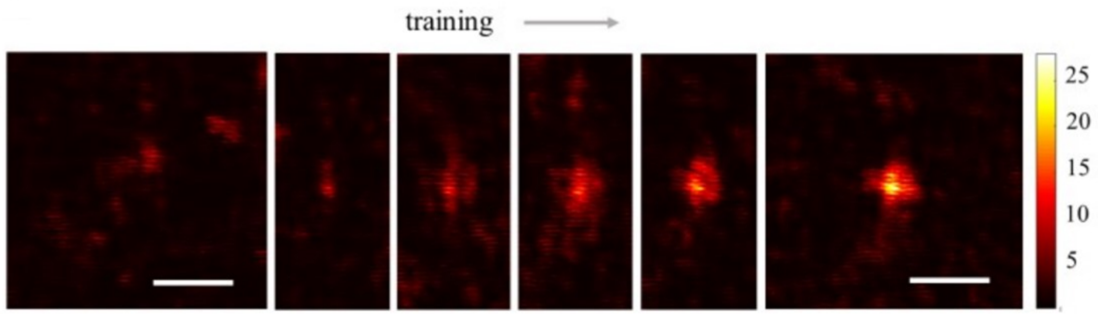
Feeback loop to find the maximal intensity (training)

The optimization of the output intensity can be found by various iterative algorithms

- sequential
- Monte Carlo
- genetic algorithms
- etc

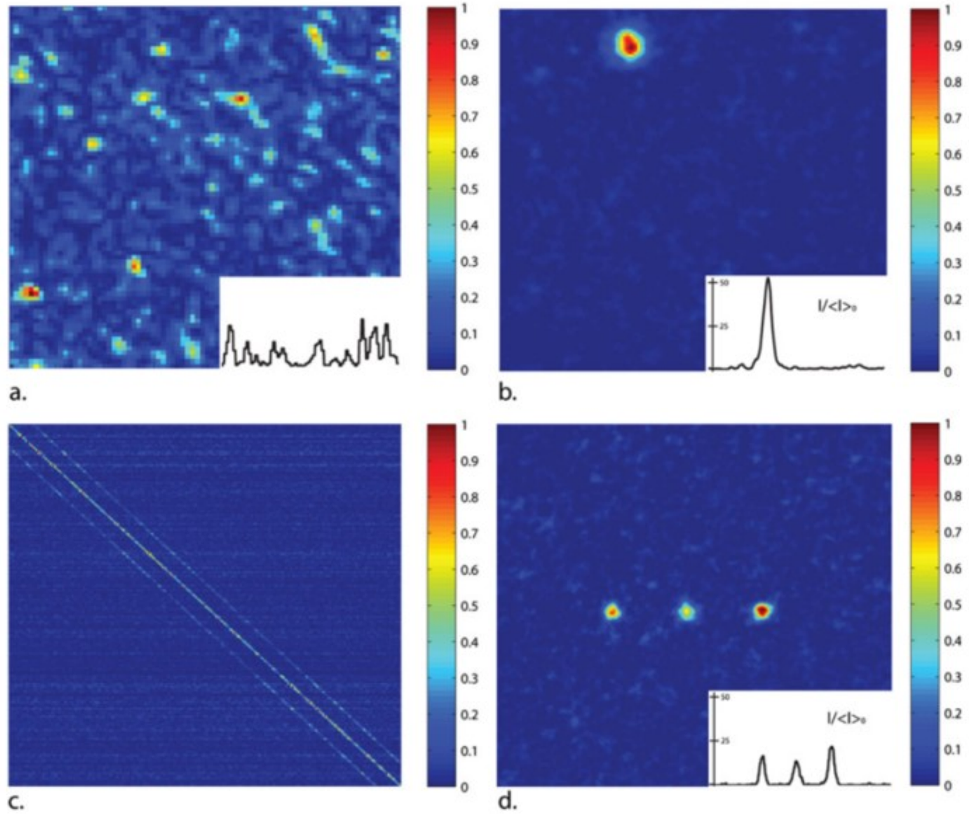


Vellekoop, 2008



Pierangeli et al, arXiv:1812.09311

Multiple point focusing and image formation



Selected for a Viewpoint in *Physics*
PRL **104,** 100601 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

Measuring the Transmission Matrix in Optics: An Approach to the Study and Control of Light Propagation in Disordered Media

S. M. Popoff, G. Lerosey, R. Carminati, M. Fink, A. C. Boccara, and S. Gigan
Institut Langevin, ESPCI ParisTech, CNRS UMR 7587, ESPCI, 10 rue Vauquelin, 75005 Paris, France
(Received 27 October 2009; revised manuscript received 11 January 2010; published 8 March 2010)

The transmission matrix in the nonlinear regime

Why ?

- Modulating the properties of the transmission of a complex system is the starting point for control and applications

  - Switching
  - Sensors
  - All-optical neural networks
  - All optical processing

Areogel: random and (thermally) nonlinear !

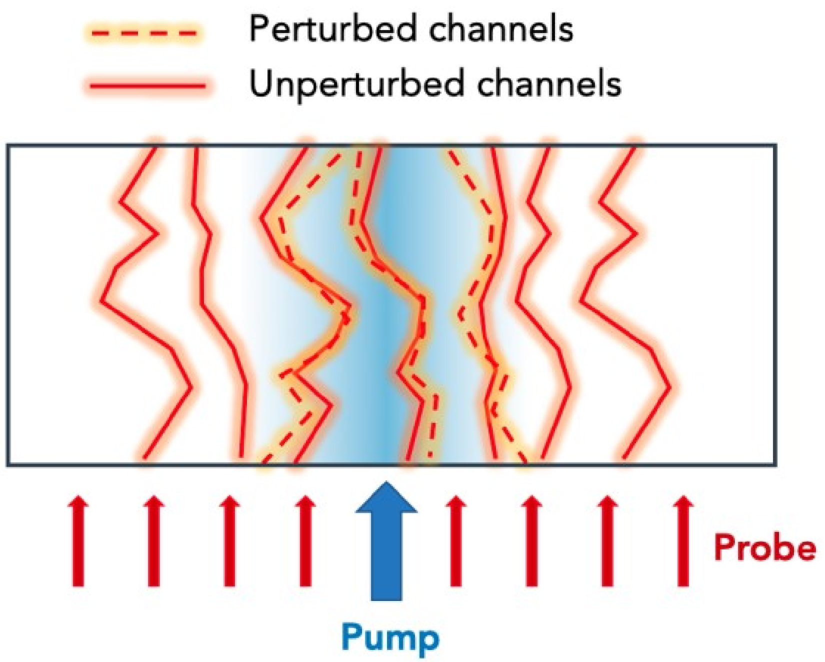


FIG. 1. Sketch of the formation dynamics of transmissive channels in a pump/probe configuration.

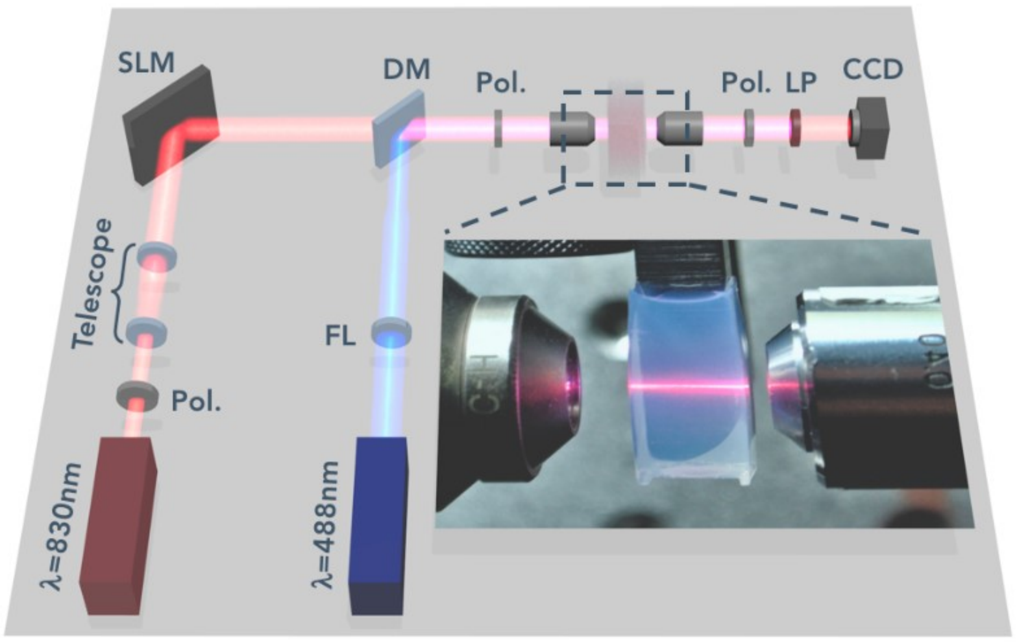


FIG. 1. Pump-Probe Optical setup with wavefront shaping of the probe beam by SLM.

Fleming, CC, Di Falco, ArXiv:1809.07077

Measure of the TM: unfolding the modes into channels

The process for forming the TM from raw image data is outlined in figure S4. The 2D pixels of the CCD (M pixels) and of the SLM (N pixels) are mapped in a MxN TM matrix. To improve the SNR in the CCD images, we sum the total black-white intensity values over 8x8 pixels, giving a measurement range between 0 and 16383, rather than 0 to 255.

The phase of each pixel of the SLM is tuned in turn in the range $(-\pi, \pi)$, keeping the other pixels at $-\pi$ and the corresponding CCD image is acquired. The light impinging on the constant area of the SLM interferes with that of the tuned pixel, to access the complex values of the transmission channel. This process produces a stack of 3D images for each SLM pixel, as shown in panel c).

The intensity of each pixel in the stack changes with the phase of the SLM pixel in a cosine function. The amplitude and phase of the relative elements of the TM are given by the peak-to-peak value of the cosine function and by the offset respect to the reference phase, respectively, as seen in panels d-e). A typical complex TM is shown in panel f).

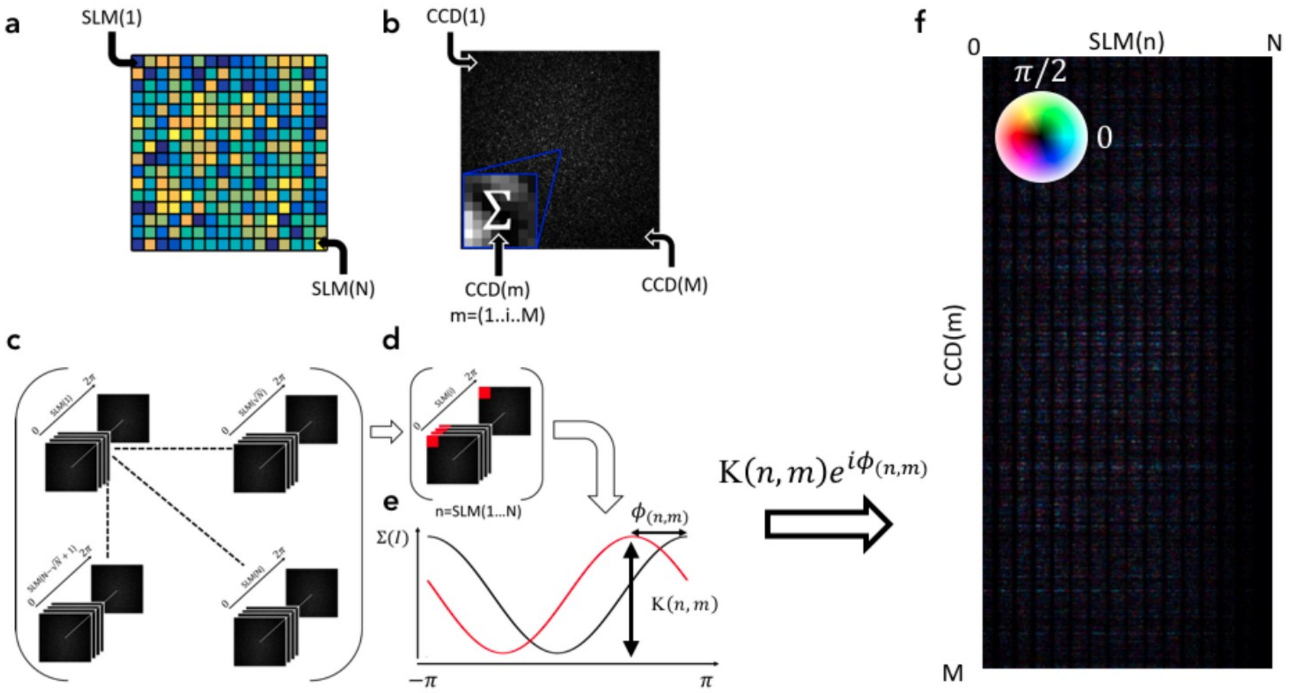
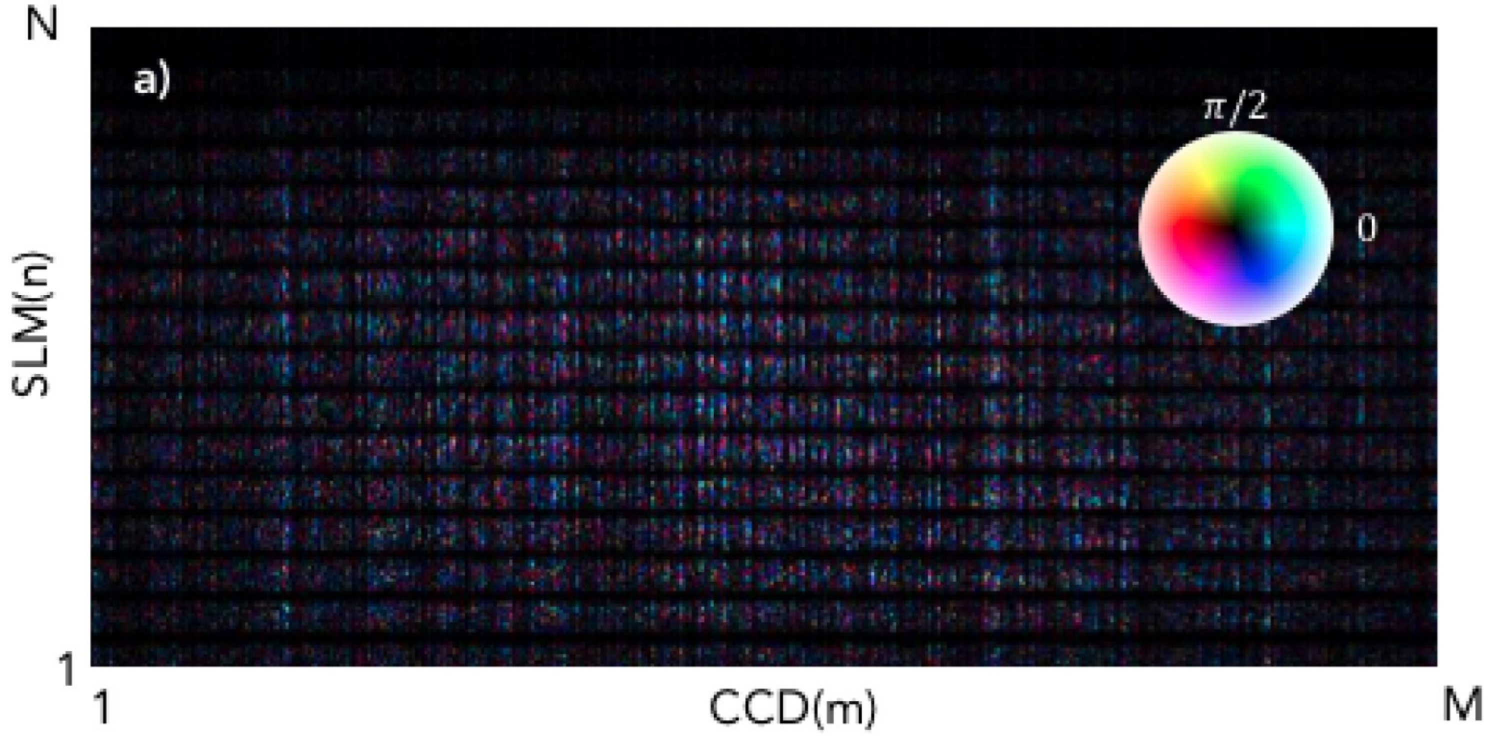


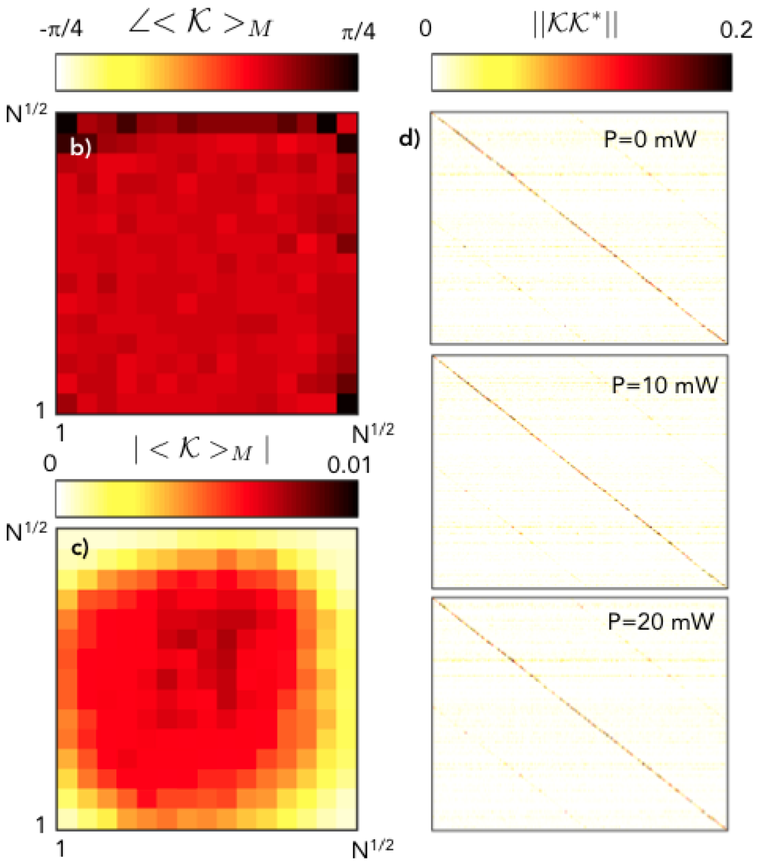
FIG. 4. Process outline for the determination of the Complex Transmission Matrices.

Fleming, CC, Di Falco, ArXiv:1809.07077

Raw data for the transmission matrix



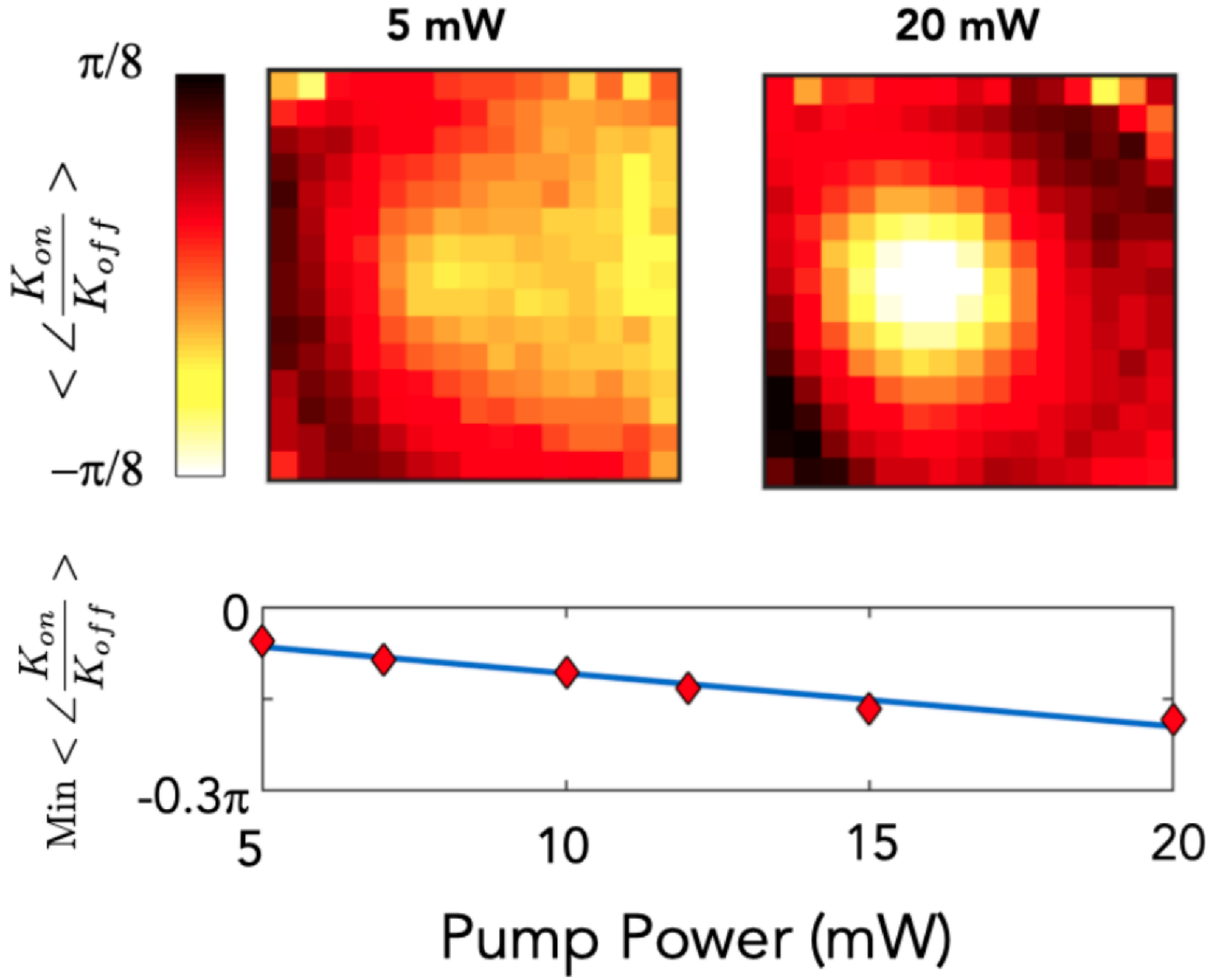
The transmission at different pump power



$$K^{\dagger} = K$$

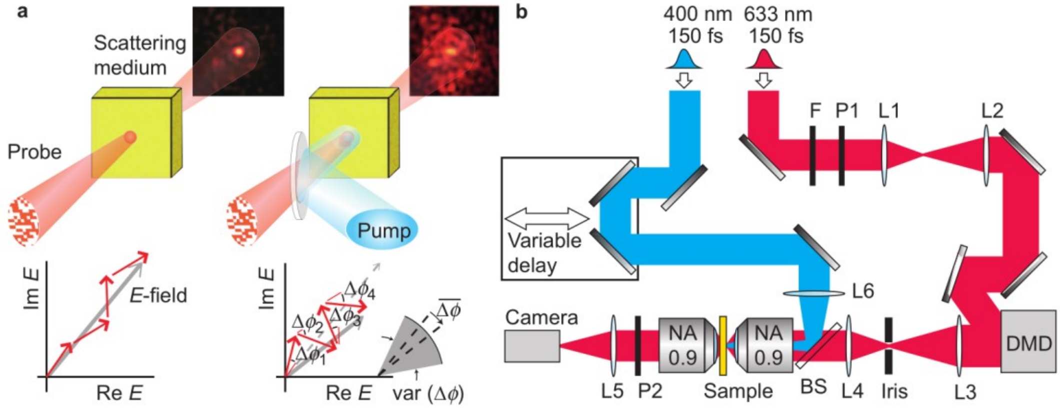
Fleming, CC, Di Falco, ArXiv:1809.07077

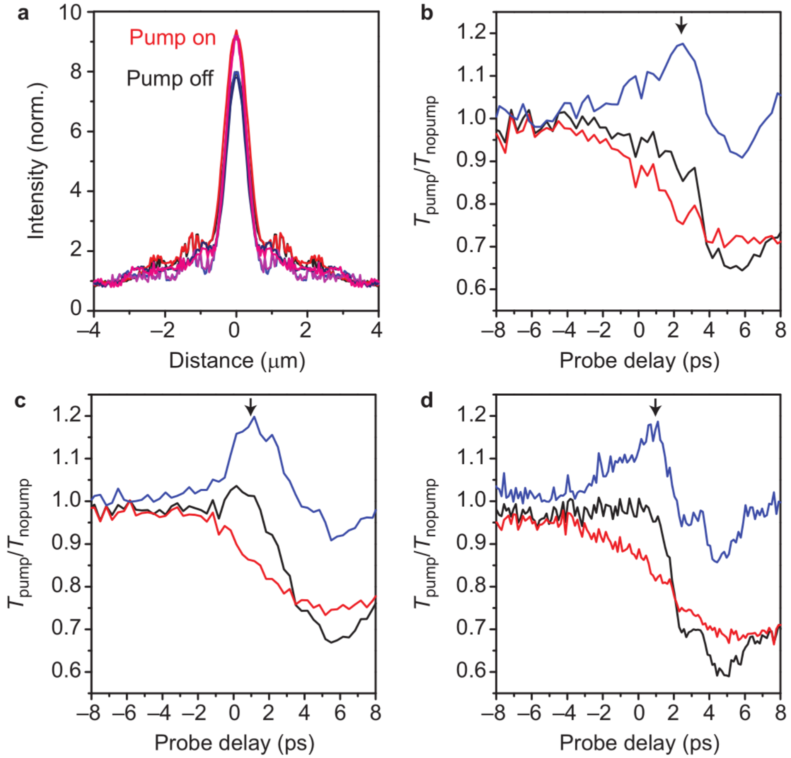
Nonlinear modulation



Fleming, CC, Di Falco, ArXiv:1809.07077

Ultrafast switching in random media





OPEN

Light: Science & Applications (2014) 3, e207; doi:10.1038/lsa.2014.88

www.nature.com/lsa

ORIGINAL ARTICLE

An ultrafast reconfigurable nanophotonic switch using wavefront shaping of light in a nonlinear nanomaterial

Tom Strudley[1], Roman Bruck[1], Ben Mills[2] and Otto L Muskens[1]

Nonlinear perturbation to the propagator

Perturbed propagator

$$(\mathcal{D} + \mathbf{e}_b)\,|\mathbf{E}_0\rangle = 0, \qquad \varepsilon_r(\mathbf{r}) = \varepsilon_b(\mathbf{r}) + \varepsilon_a(\mathbf{r})$$

$$(\mathcal{D} + \mathbf{e}_b + \mathbf{e}_s)\,|\mathbf{E}\rangle = 0 \qquad |\mathbf{E}\rangle = \mathbf{K}|\mathbf{E}_0\rangle$$

$$\varepsilon_r(\mathbf{r}) = \varepsilon_b(\mathbf{r}) + \varepsilon_a(\mathbf{r}) + \Delta\varepsilon(\mathbf{r})$$

$$(\mathcal{D} + \mathbf{e}_b + \mathbf{e}_s + \mathbf{e}')\,|\mathbf{E}'\rangle = 0$$

$$|\mathbf{E}'\rangle = \mathbf{K}'|\mathbf{E}\rangle = \mathbf{K}'\mathbf{K}|\mathbf{E}_0\rangle.$$

$$\mathbf{K}' = 1 - \mathbf{G}'\mathbf{e}'$$

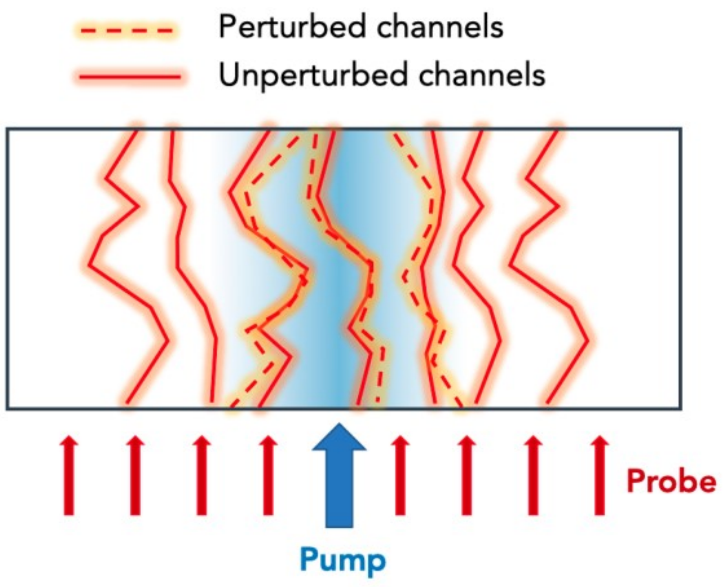


FIG. 1. Sketch of the formation dynamics of transmissive channels in a pump/probe configuration.

Nonlinear perturbation as a new learning level

$$|\mathbf{E}'\rangle = \mathbf{K}'|\mathbf{E}\rangle = \mathbf{K}'\mathbf{K}|\mathbf{E}_0\rangle.$$

$$k_{mn}^{\mathrm{NL}} = k'_{mq}k_{qn}$$

$$\mathbf{K}' = \mathbf{1} - \mathbf{G}'\mathbf{e}'$$

$$k'_{mq} = \delta_{mq} + w_{mq},$$

$$w_{mq} = -\langle m|\mathbf{G}'\mathbf{e}'|n\rangle.$$

$$k_{mn}^{\mathrm{NL}} = k_{mn} + w_{mq}k_{qn} = k_{mn} + w_{m1}k_{1n} + \ldots + w_{mN}k_{Nn}.$$

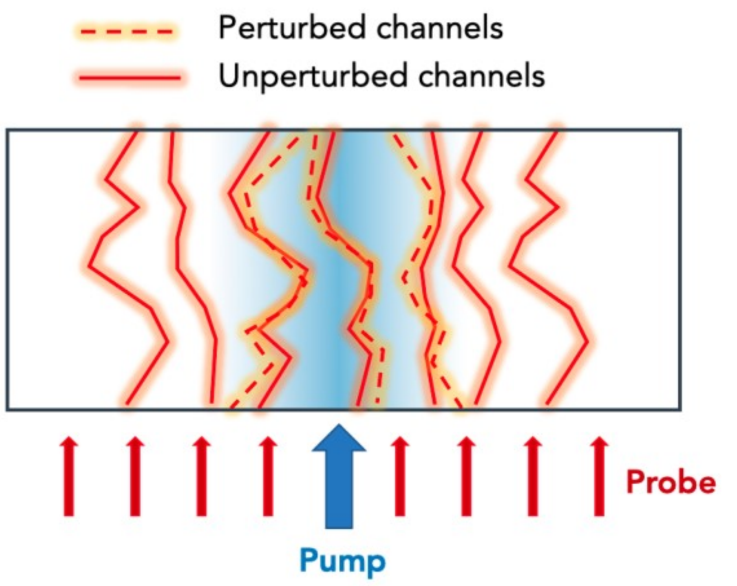


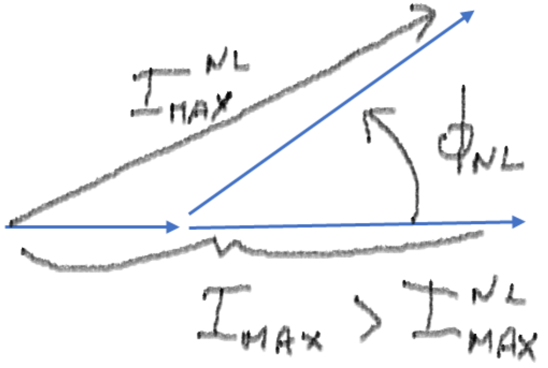
FIG. 1. Sketch of the formation dynamics of transmissive channels in a pump/probe configuration.

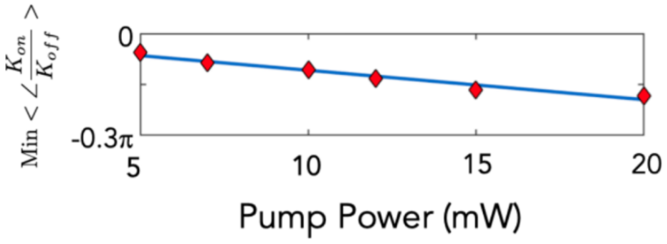
The effect of the perturbation on the focusing

$$k_{mn}^{\mathrm{NL}} = k_{mn} + w_{mq}k_{qn} = k_{mn} + w_{m1}k_{1n} + \ldots + w_{mN}k_{Nn}.$$

$$\langle |k_{mn}^{\mathrm{NL}}|^2 \rangle = \langle |k_{mn}|^2 \rangle.$$

$$k_{mn}^{\mathrm{NL}} = k_{mn}\frac{1+\xi_{mn}}{\sqrt{1+2\phi_{NL}^2}}, = k_{mn}e^{\imath\kappa_{mn}\phi_{NL}}$$





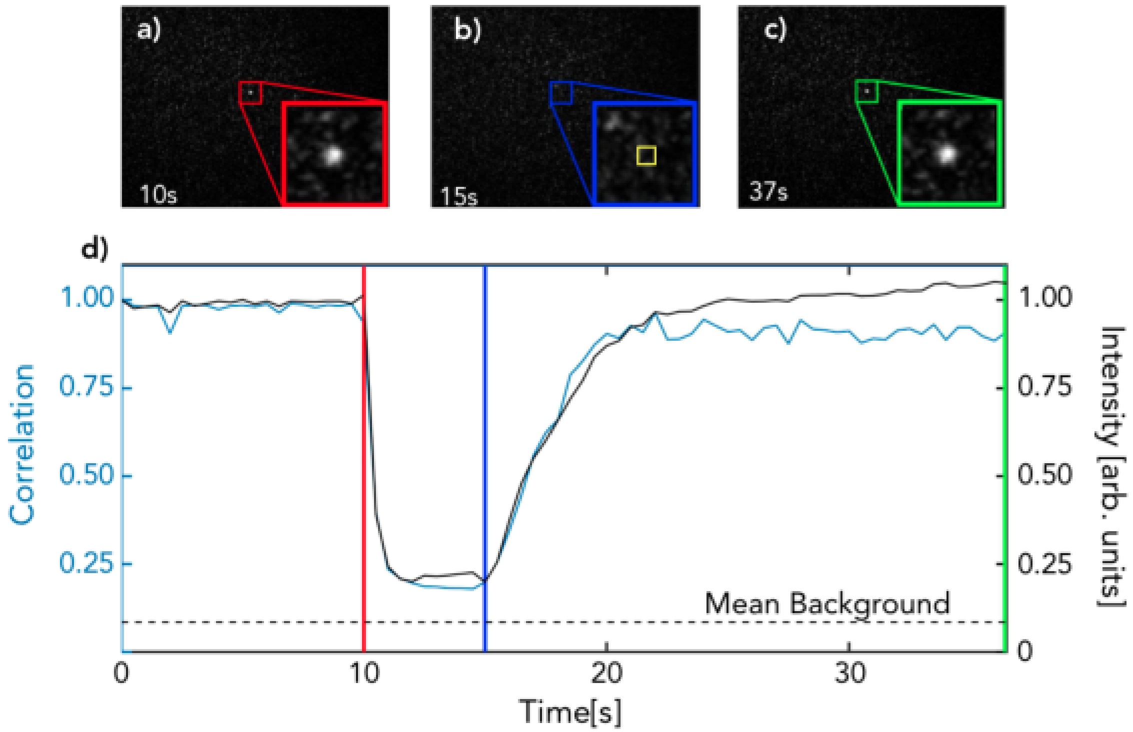
$$\boxed{\eta^{NL} = \frac{\langle I_{MAX} \rangle}{\langle I_0 \rangle} \cong \eta \ \left(1-\phi_{NL}^2\right)}$$

$$\phi_{NL} \simeq \frac{\pi\omega}{2}\sqrt{\langle | \int \Delta\varepsilon(\mathbf{r})\rho(\mathbf{r},\omega)\mathrm{d}\mathbf{r}|^2 \rangle}$$

Fleming, CC, Di Falco, ArXiv:1809.07077

Effect of the perturbation on the focusing

The nonlinear refractive perturbation reduces the enhancement



Fleming, CC, Di Falco, ArXiv:1809.07077

Application in biophysics

Tumor morphodynamics

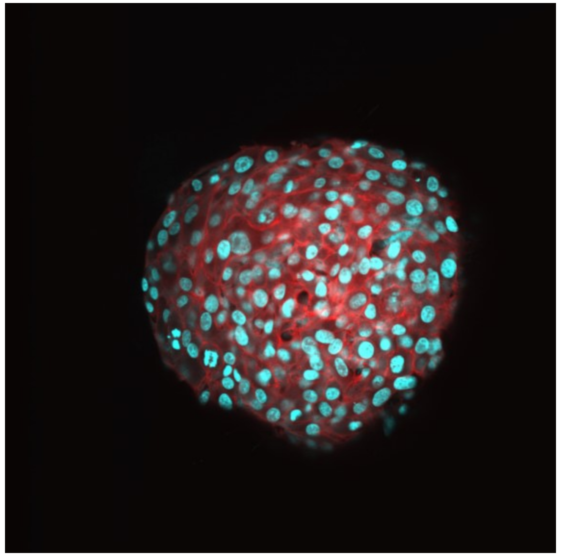
Light propagation in living (!) tumor models

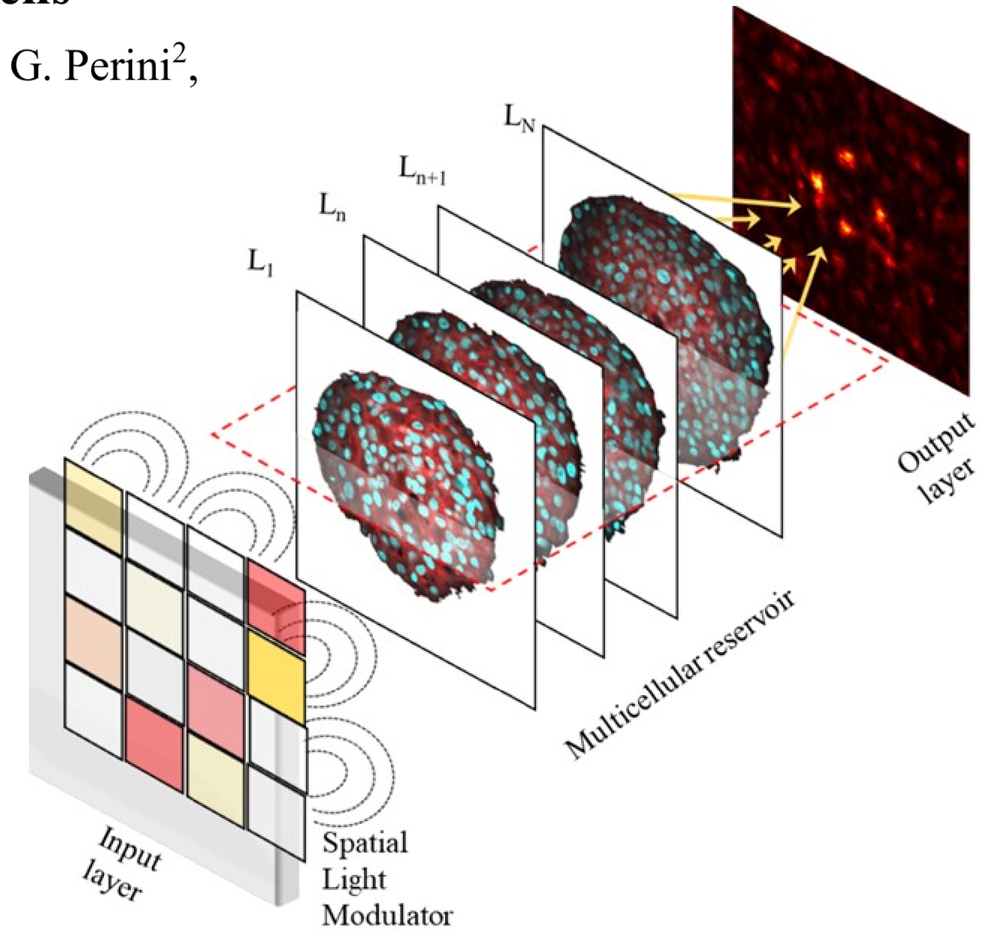
Deep optical neural network by living tumour brain cells

Authors: D. Pierangeli[1,4]†, V. Palmieri[2,4]†, G. Marcucci[1,4], C. Moriconi[3], G. Perini[2], M. De Spirito[2], M. Papi[2]*, C. Conti[1,4]*

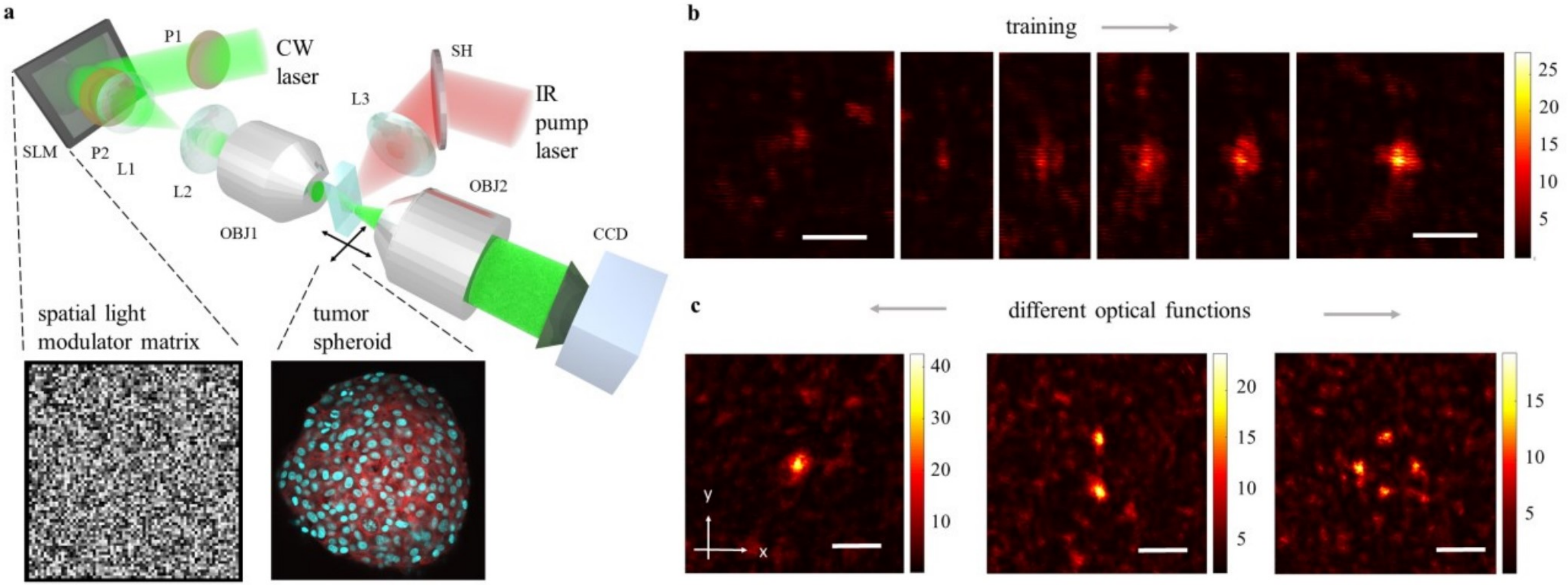
ArXiv:1812.09311

Glioblastoma cells forming a spheroidal cancer model.

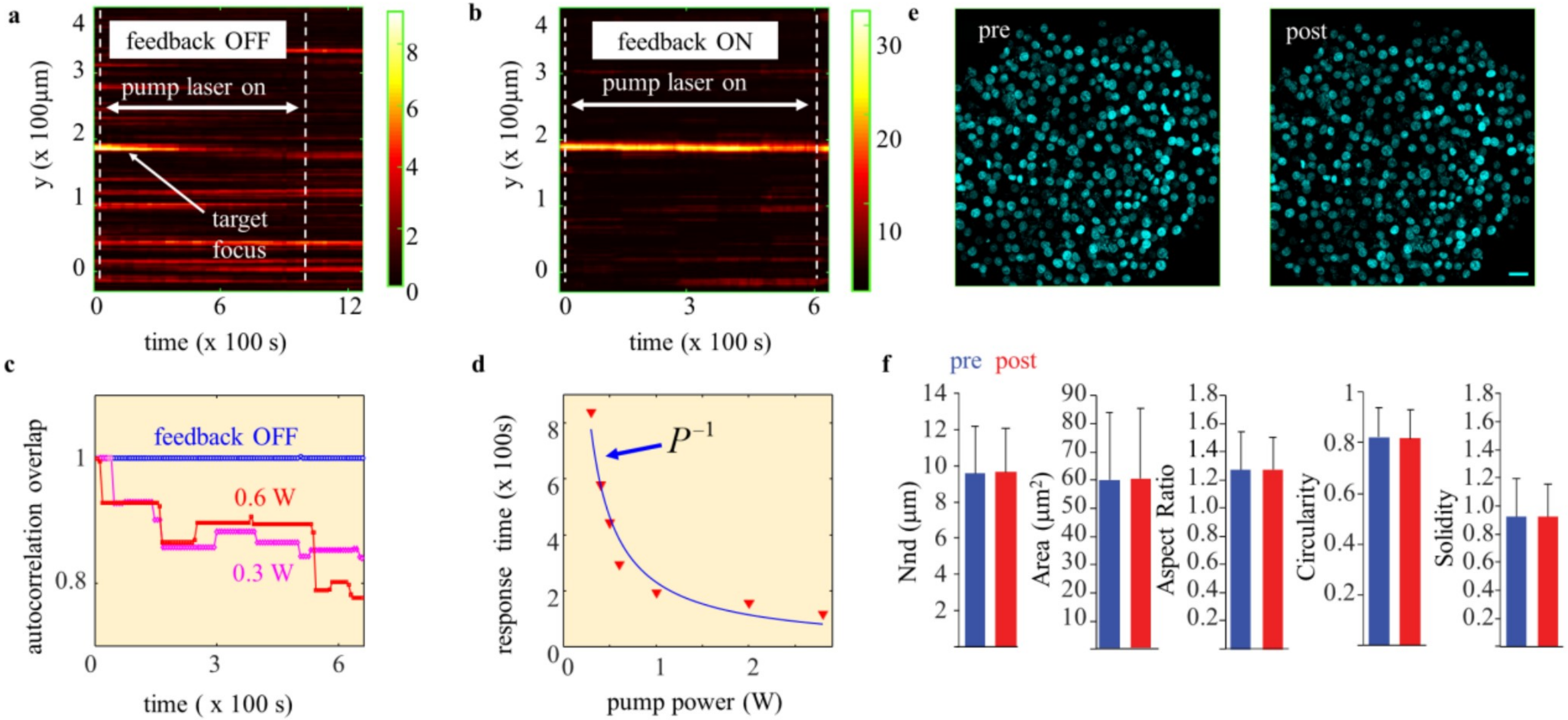




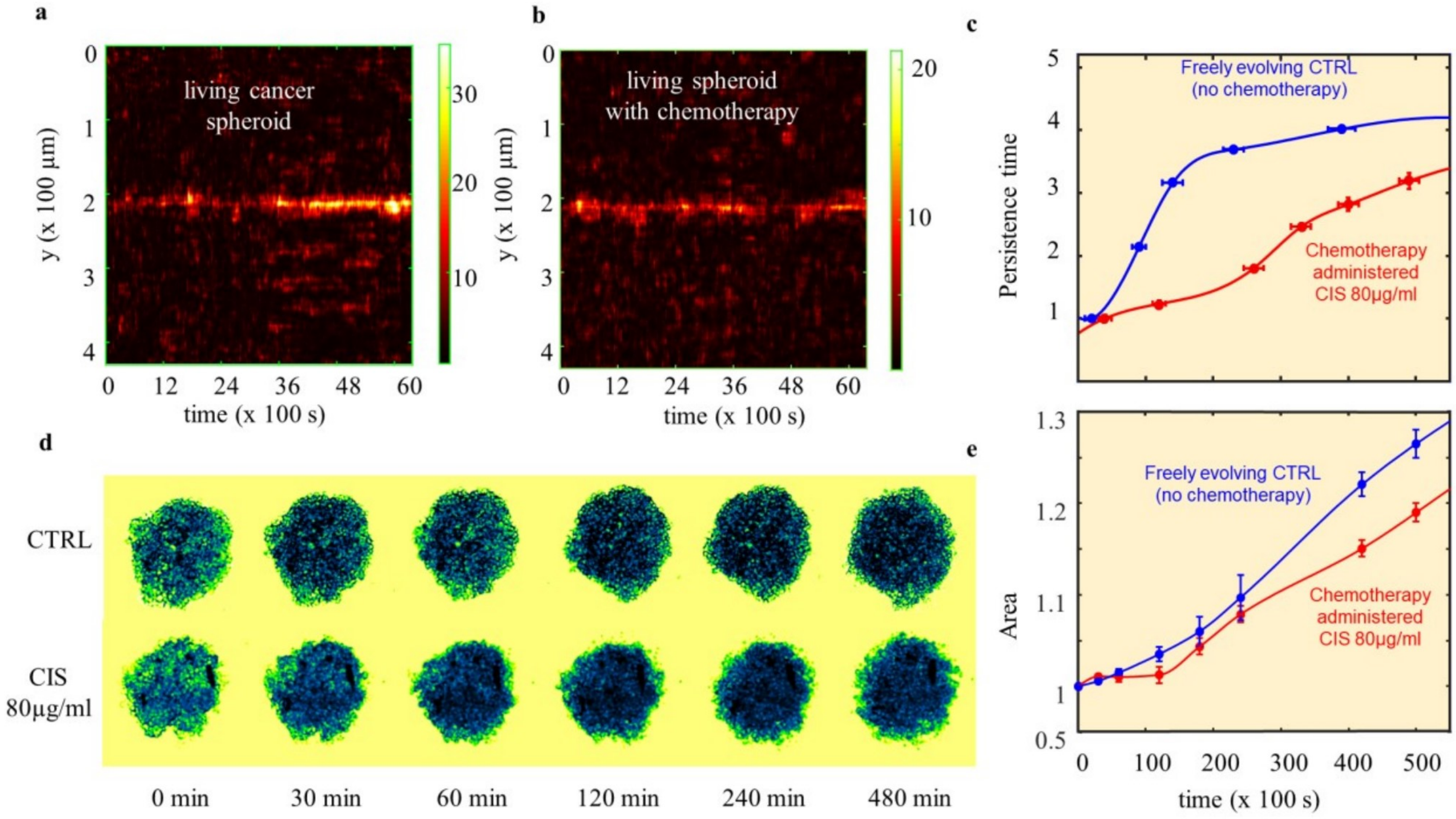
Training the light transmission



Nonlinear perturbation



Chemical perturbation (chemotherapy)



Optical neural networks

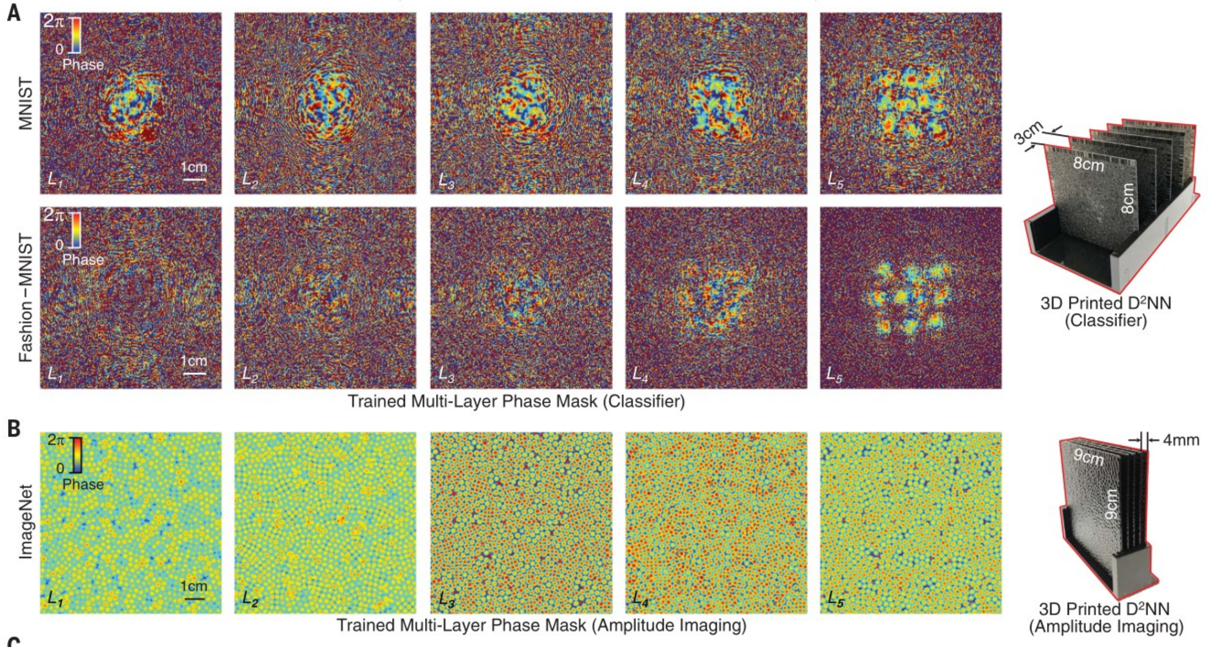
RESEARCH

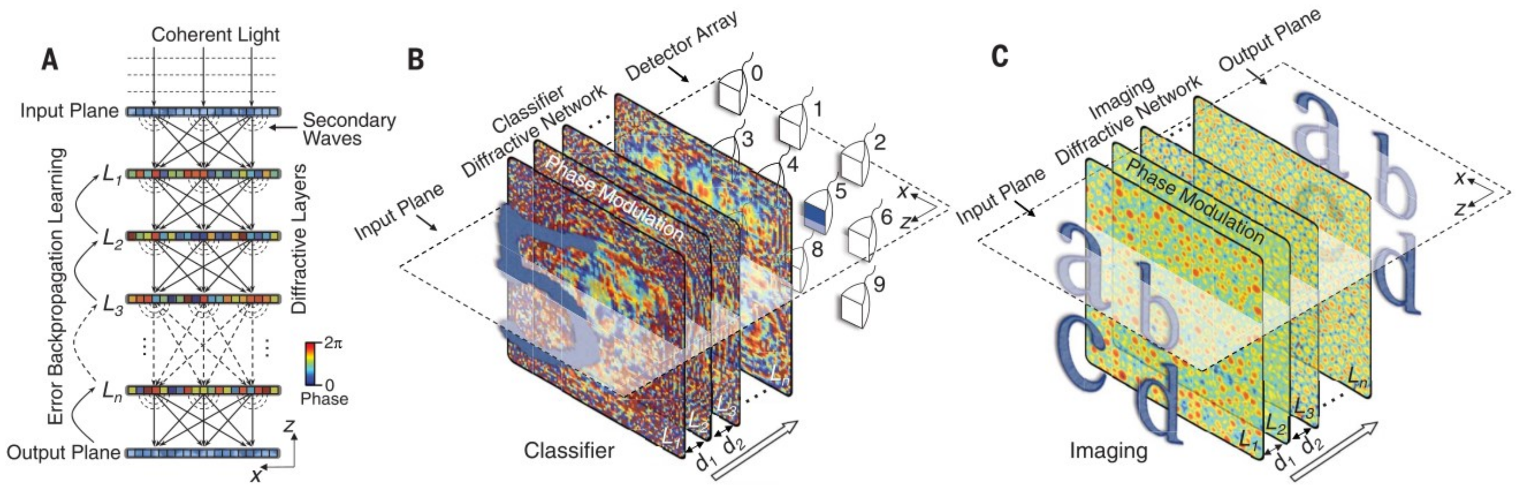
OPTICAL COMPUTING

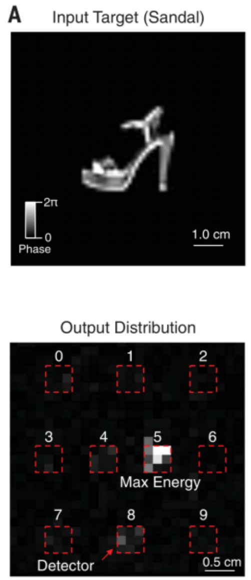
All-optical machine learning using diffractive deep neural networks

Xing Lin[1,2,3]*, Yair Rivenson[1,2,3]*, Nezih T. Yardimci[1,3], Muhammed Veli[1,2,3], Yi Luo[1,2,3], Mona Jarrahi[1,3], Aydogan Ozcan[1,2,3,4]†

Lin *et al.*, *Science* **361**, 1004–1008 (2018) 7 September 2018







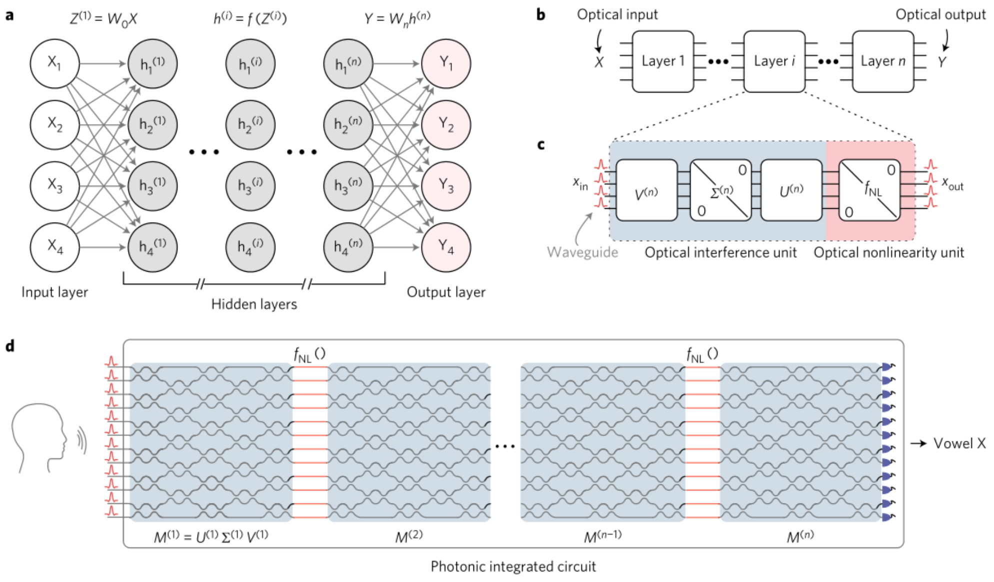
Fashion classifier

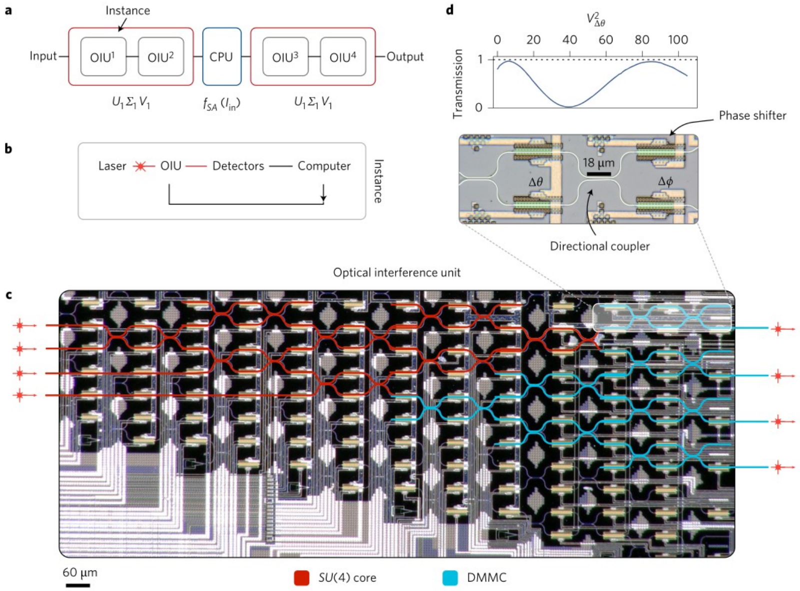
NLP2019

nature photonics
ARTICLES
PUBLISHED ONLINE: 12 JUNE 2017 | DOI: 10.1038/NPHOTON.2017.93

Deep learning with coherent nanophotonic circuits

Yichen Shen[1*†], Nicholas C. Harris[1*†], Scott Skirlo[1], Mihika Prabhu[1], Tom Baehr-Jones[2], Michael Hochberg[2], Xin Sun[3], Shijie Zhao[4], Hugo Larochelle[5], Dirk Englund[1] and Marin Soljačić[1]





Other applications

- Ising machine and combinatorial problems
- Random lasers
- Quantum gates and quantum cryptography
- ….

NLP2019

Deadline 5 feb 2019

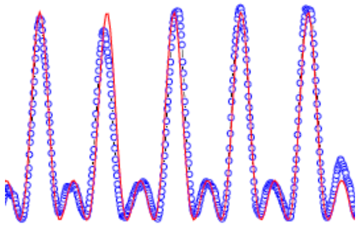
Nonlinear Optics

- Nail Akhmediev, *Australian National University, Australia*
- Jens Biegert, *ICFO -Institut de Ciencies Fotoniques, Spain*
- John Bowers, *University of California Santa Barbara, United States*
- Daniel Brunner, *CNRS, France*
- Hui Cao, *Yale University, United States*
- Demetrios Christodoulides, *University of Central Florida, United States*
- Majid Ebrahim-Zadeh, *ICFO -Institut de Ciencies Fotoniques, Spain*
- Miro Erkintalo, *University of Auckland, New Zealand*
- Shanhui Fan, *Stanford University, United States*
- Mark Foster, *Johns Hopkins University, United States*
- Rupert Huber, *Universität Regensburg, Germany*
- Franz Kaertner, *Center for Free Electron Laser Science, Germany*
- Tobias Kippenberg, *Ecole Polytechnique Federale de Lausanne, Switzerland*
- Yuri Kivshar, *Australian National University, Australia*
- J. Kutz, *University of Washington, United States*
- Marko Loncar, *Harvard University, United States*
- Kathy Lüdge, *Technische Universität Berlin, Germany*
- Alexander Lukin, *Harvard University*
- Alireza Marandi, *California Institute of Technology, United States*
- Alessia Pasquazi, *University of Sussex, United Kingdom*
- Antonio Picozzi, *Centre National Recherche Scientifique, France*
- Peter Rakich, *Yale University, United States*

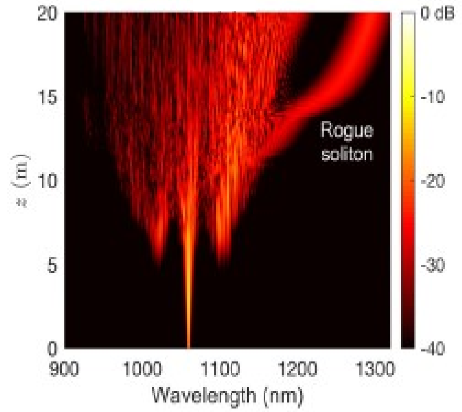
Dynamical complexity

Nonperturbative nonlinearity

Fermi-Pasta-Ulam-Tsinguo



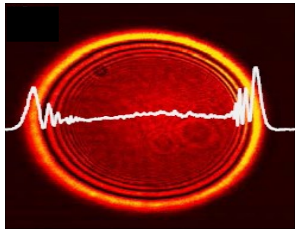
Supercontinuum



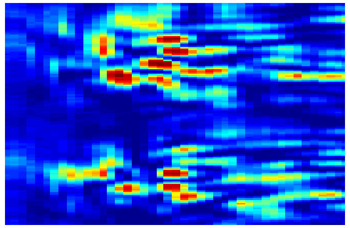
Anderson localization

Rogue waves

Shock waves



Optical turbulence



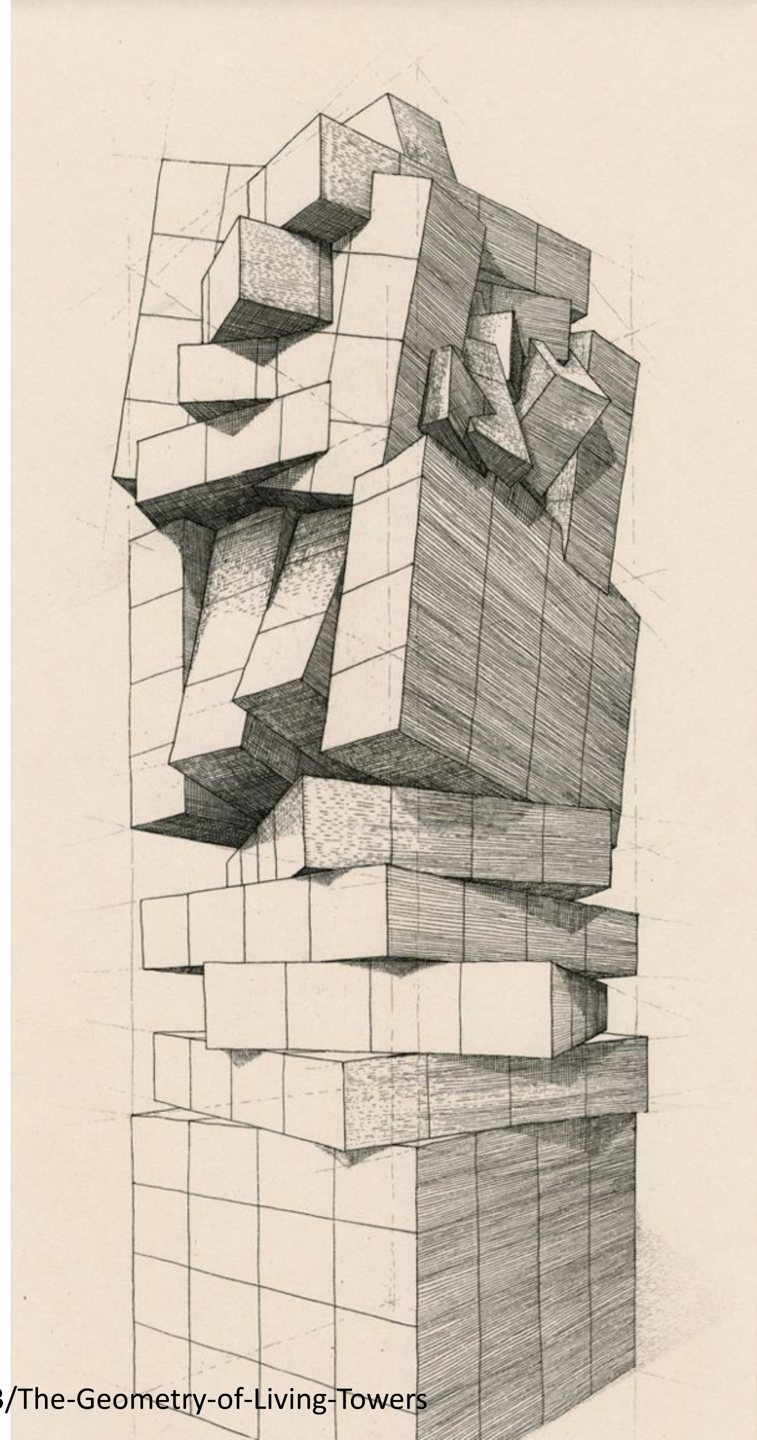
Condensation

Beam-cleaning

Simple Vs Complex

The number of «states» (linear or nonlinear) is a simple way to distinguish simple and complex scenarios

This is related to the amount of information you need for any mathematical description of the system



https://www.behance.net/gallery/6909553/The-Geometry-of-Living-Towers

States due to nonlinearity?

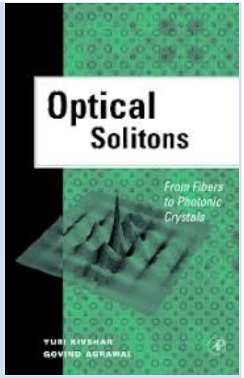
…. solitons ….

A simple model for
complex dynamics

The nonlinear Schroedinger equation

The NLS from nonlinear Maxwell equations

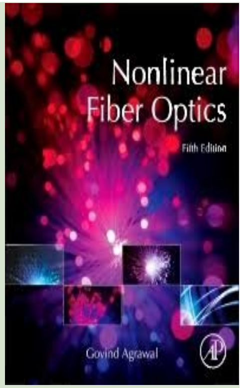
1



Spatial case

$$2ik\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} + 2k^2\frac{n_2|A|^2}{n_0}A = 0$$

2



Temporal case

$$i\frac{\partial A}{\partial z} + \frac{i\alpha}{2}A - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma|A|^2 A = 0.$$

Souls of the NLS (focusing)

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2 u=0,$$

Simple normalized NLS equation
(fundamental soliton, supercontinuum, and related)

$$i\varepsilon\psi_t+\frac{\varepsilon^2}{2}\psi_{xx}+\psi|\psi|^2=0,$$

NLS in the hydrodynamic regime
(rogue waves, shocks, FPU, and complex wave regimes)

$$\imath\partial_t\hat{\phi}=-\hat{\phi}_{xx}+2c\hat{\phi}^\dagger\hat{\phi}\hat{\phi}$$

Second quantized NLS
(quantum soliton, squeezing, and all of that)

NLS full optional

$$i\frac{\partial U}{\partial Z} + \sum_{k\geq 2} \frac{i^k}{k!}\beta_k \frac{\partial^k U}{\partial T^k}$$

$$+ \gamma\left(1 + i\tau_{\text{shock}}\frac{\partial}{\partial T}\right) U \int_0^\infty R(T')|U(T - T')|^2 \mathrm{d}T' = 0,$$

Souls of the NLS (focusing)

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0,$$

NONLINEARITY = DISPERSION

$$\psi = u / \varepsilon$$

$$i\varepsilon\psi_t + \frac{\varepsilon^2}{2}\psi_{xx} + \psi|\psi|^2 = 0,$$

NONLINEARITY >> DISPERSION

$$\imath\partial_t\hat{\phi} = -\hat{\phi}_{xx} + 2c\hat{\phi}^\dagger\hat{\phi}\hat{\phi}$$

?????

Souls of the NLS (focusing)

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0,$$

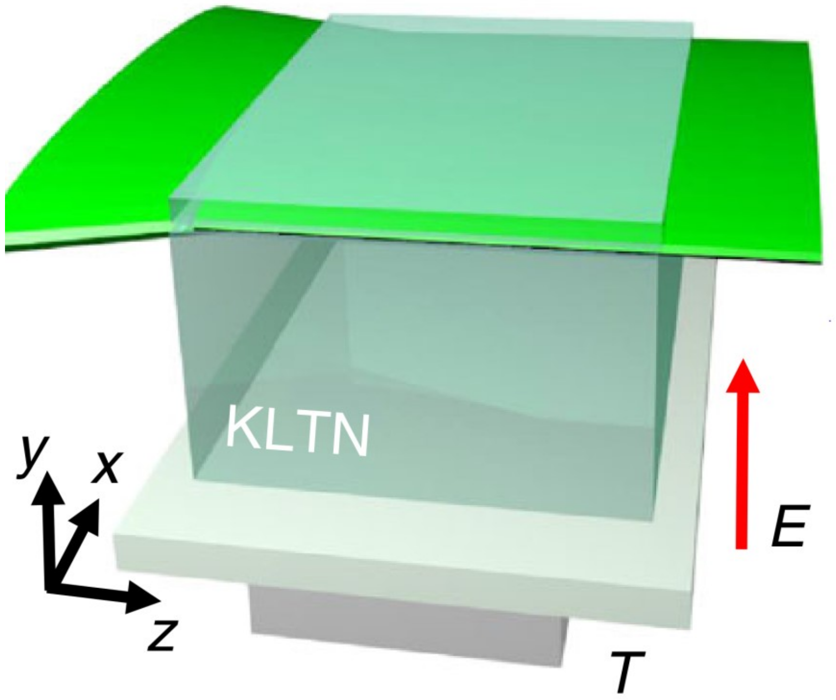
>10000 published papers

$$i\varepsilon\psi_t + \frac{\varepsilon^2}{2}\psi_{xx} + \psi|\psi|^2 = 0,$$

100-1000 published papers

$$\imath\partial_t\hat{\phi} = -\hat{\phi}_{xx} + 2c\hat{\phi}^\dagger\hat{\phi}\hat{\phi}$$

10-100 published papers



Simple derivation of NLS

Spatial case

From scratch …

The wave equation (scalar is enough)

$$\nabla^2 \mathcal{E} - \frac{n^2}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$$

Time harmonic field

$$\mathcal{E} = E\cos(\omega t - kz) = \Re[Ee^{-i\omega t + ikz}]$$

$$k = \frac{\omega n_0}{c} = \frac{2\pi n_0}{\lambda}$$

Helmholtz equation

$$\mathcal{E} = \Re[E(x,y,z)e^{-i\omega t + ikz}]$$

$$\nabla^2 E + \omega^2 \frac{n^2}{c^2} E = 0$$

$$I = \frac{c\epsilon_0}{2} |E|^2 = |A|^2$$

The nonlinear refractive index

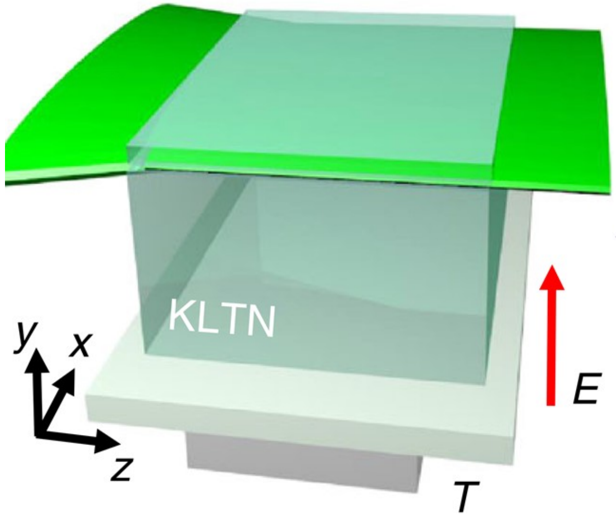
$$n = n_0 + \Delta n[|A|^2] = n_0 + \Delta n[I]$$

$$\Delta n = n_2 I$$

The paraxial approximation

$$\cancel{\frac{\partial^2 A}{\partial z^2}} + 2ik\frac{\partial A}{\partial z} + \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) + 2k^2\frac{\Delta n}{n_0} = 0$$

$$\partial_y A = 0$$

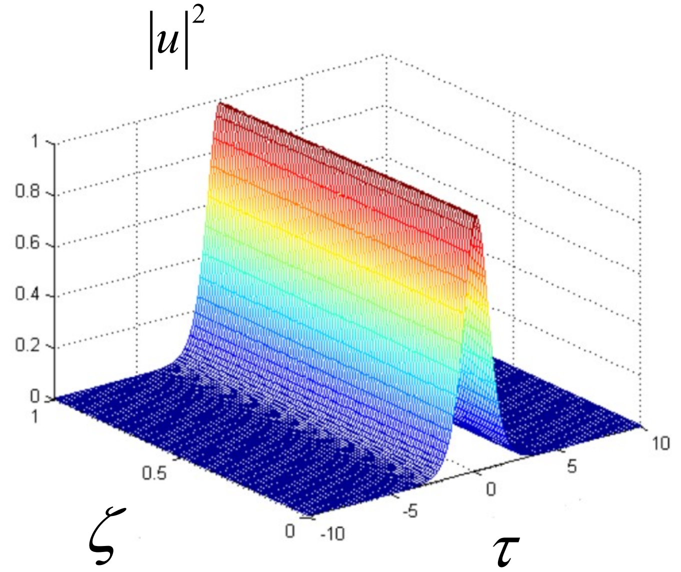


$$2\imath k\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} + 2k^2\frac{n_2|A|^2}{n_0}A = 0$$

Normalization and single soliton solution

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0,$$

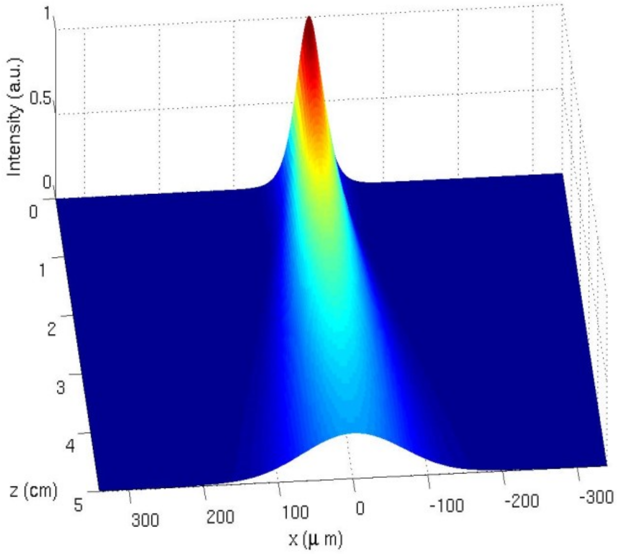
$$u(\xi,\tau) = \eta\,\mathrm{sech}[\eta(\tau - \tau_s + \delta\xi)]\exp[i(\eta^2 - \delta^2)\xi/2 - i\delta\tau + i\phi_s],$$



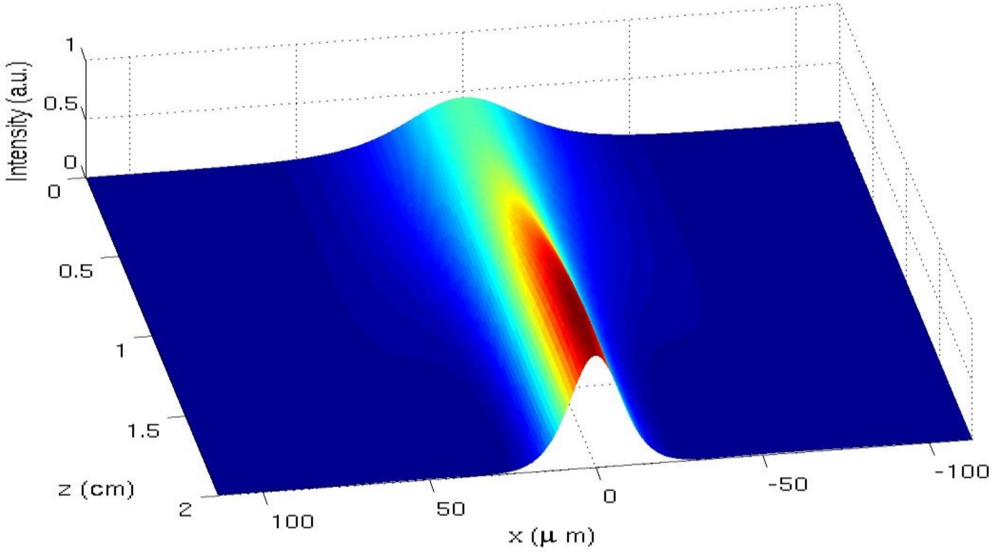
Diffraction (or dispersion) and self-trapping

Beams tend to delocalize (spread) in space

Nonlinear effects trigger self-trapping



Low intesity = diffraction

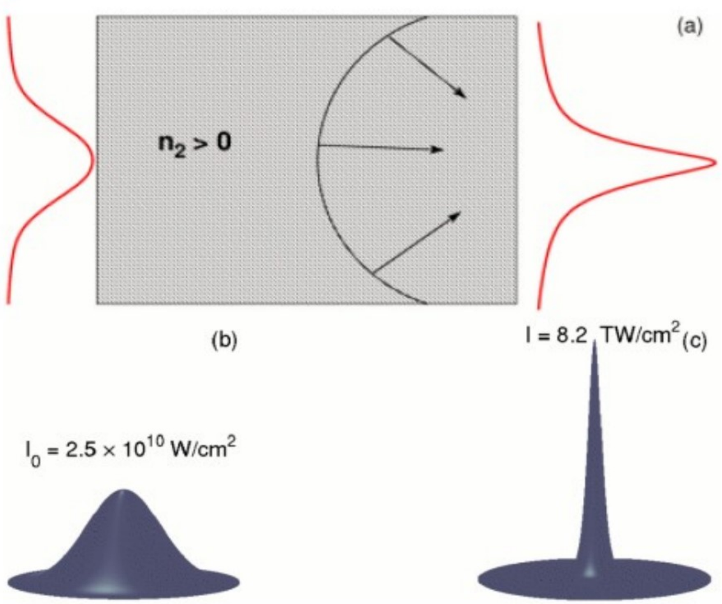


High intensity = self-trapping

The origin of the self-trapping

Refractive index $n = n_0 + n_2 I$

$n_2>0$: focusing
$n_2<0$: defocusing



Berge' et al, physics/0612063

Numerical solution of the NLS

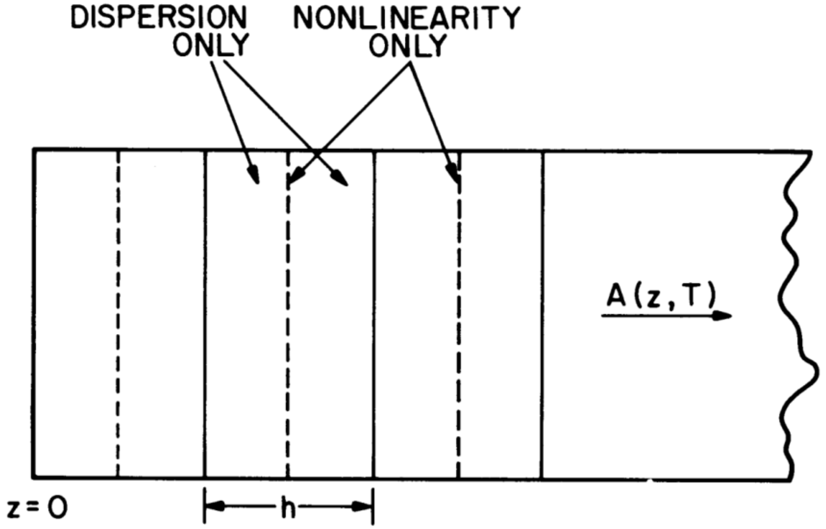
The beam propagation method

The split step method

$$i\frac{\partial\psi}{\partial z}+\frac{\partial^2\psi}{\partial x^2}+|\psi|^2\psi=0 \qquad \frac{\partial\psi}{\partial z}=i\frac{\partial^2\psi}{\partial x^2}+i|\psi|^2\psi$$

$$\frac{\partial A}{\partial z}=(\hat{D}+\hat{N})A$$

$$A(z+h,T)\approx\exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_z^{z+h}\hat{N}(z')\,dz'\right)\exp\left(\frac{h}{2}\hat{D}\right)A(z,T$$



Matlab program for the split step

Solution of the NLS by the split-step method

By Claudio, January 2019

```
close all
clearvars
```

Grid Parameters

```
Deltax=10;                    % windows
nplot=200;                    % plot number
nz=1000;                              % step between plots
zmax=1;                       % length
nx=4096;                       % number of points along x
```

Initial condition parameters

```
N=10; % soliton number
```

Grid definition

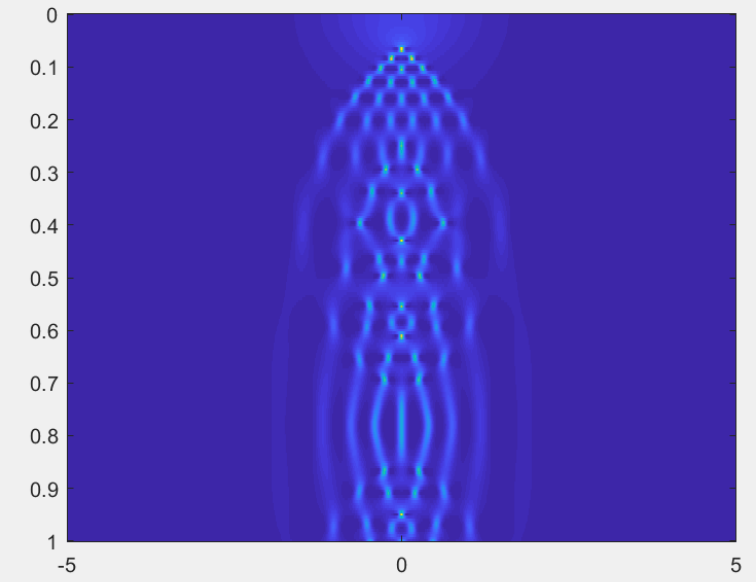
```
zstep=zmax/(nz-1)/(nplot-1);
dfx=1/Deltax;
xmin=-Deltax/2;

%point alongz  per plot
zplot=linspace(0,zmax,nplot+1);

%point in x
ix=1:nx; x=xmin+(ix-1)*Deltax/(nx-1); xs=(ix-nx/2)/Deltax;
```

Propagator for the linear part

```
ntx=0;
xx=zeros(nx,1);
```



The soliton effect compressor

6.3 Soliton-Effect Compressors

Optical pulses at wavelengths exceeding 1.3 μm generally experience both SPM and anomalous GVD during their propagation in silica fibers. Such a fiber can act as a compressor by itself without the need of an external grating pair and has been used since 1983 for this purpose [74]–[93]. The compression mechanism is related to a fundamental property of higher-order solitons. As discussed in Section A.5.2, these solitons follow a periodic evolution pattern such that they undergo an initial narrowing phase at the beginning of each

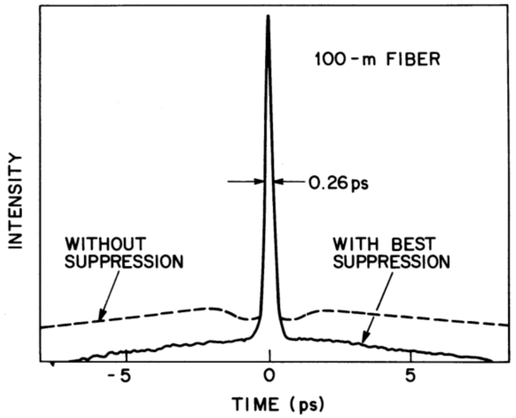
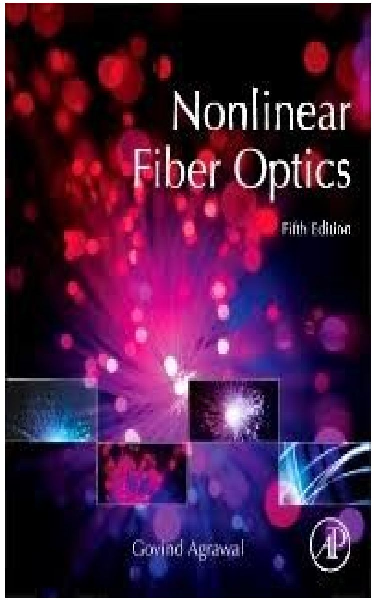
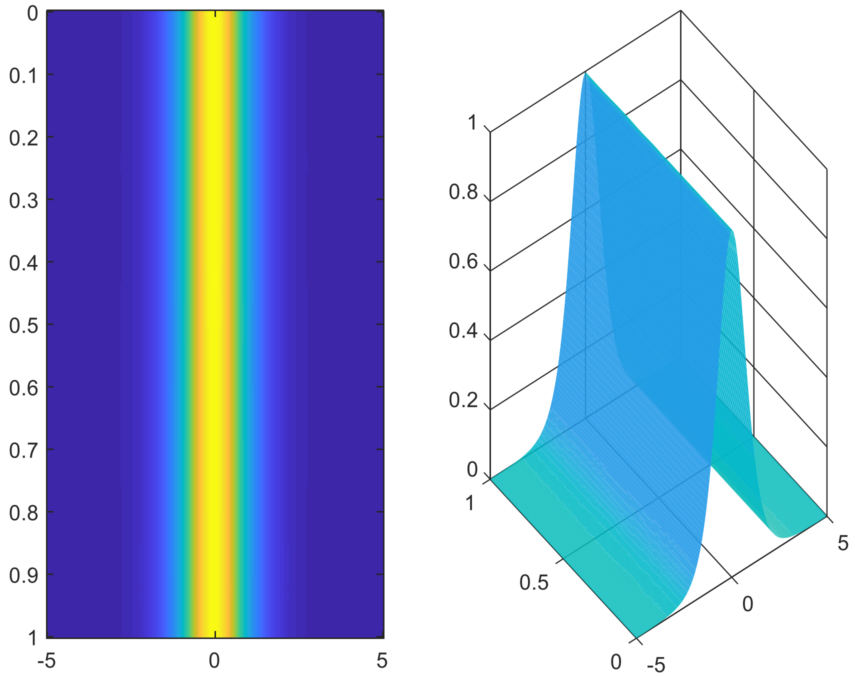


Figure 6.9 Autocorrelation trace of a 7-ps input pulse compressed to 0.26 ps by using a soliton-effect compressor. Dashed and solid curves compare the pedestal with and without the nonlinear birefringence effect. (After Ref. [74])



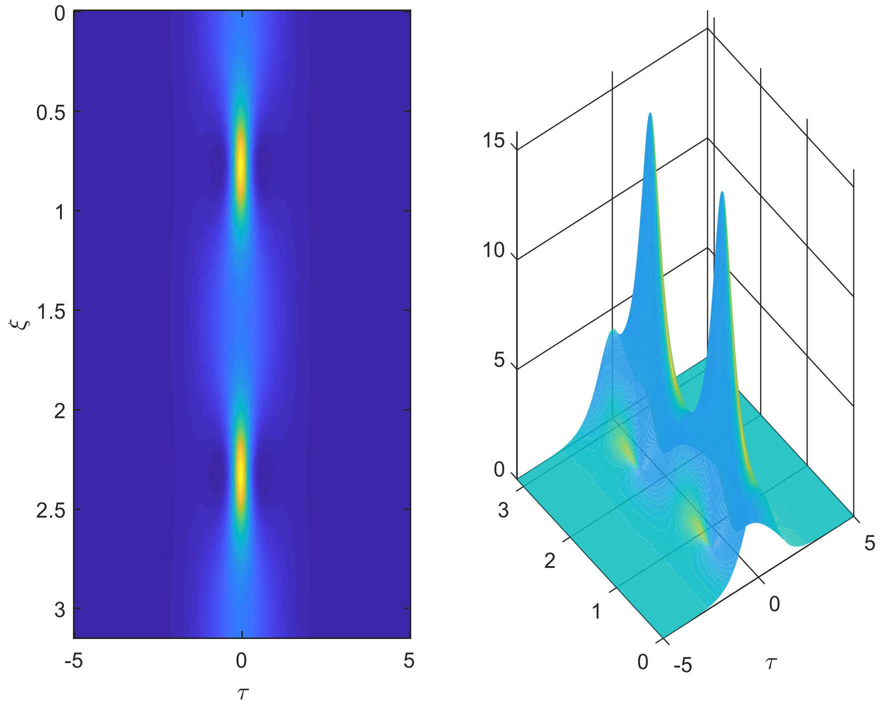
Simulation of the fundamental soliton

$$\psi(\tau,0)=\mathrm{sech}(x)$$

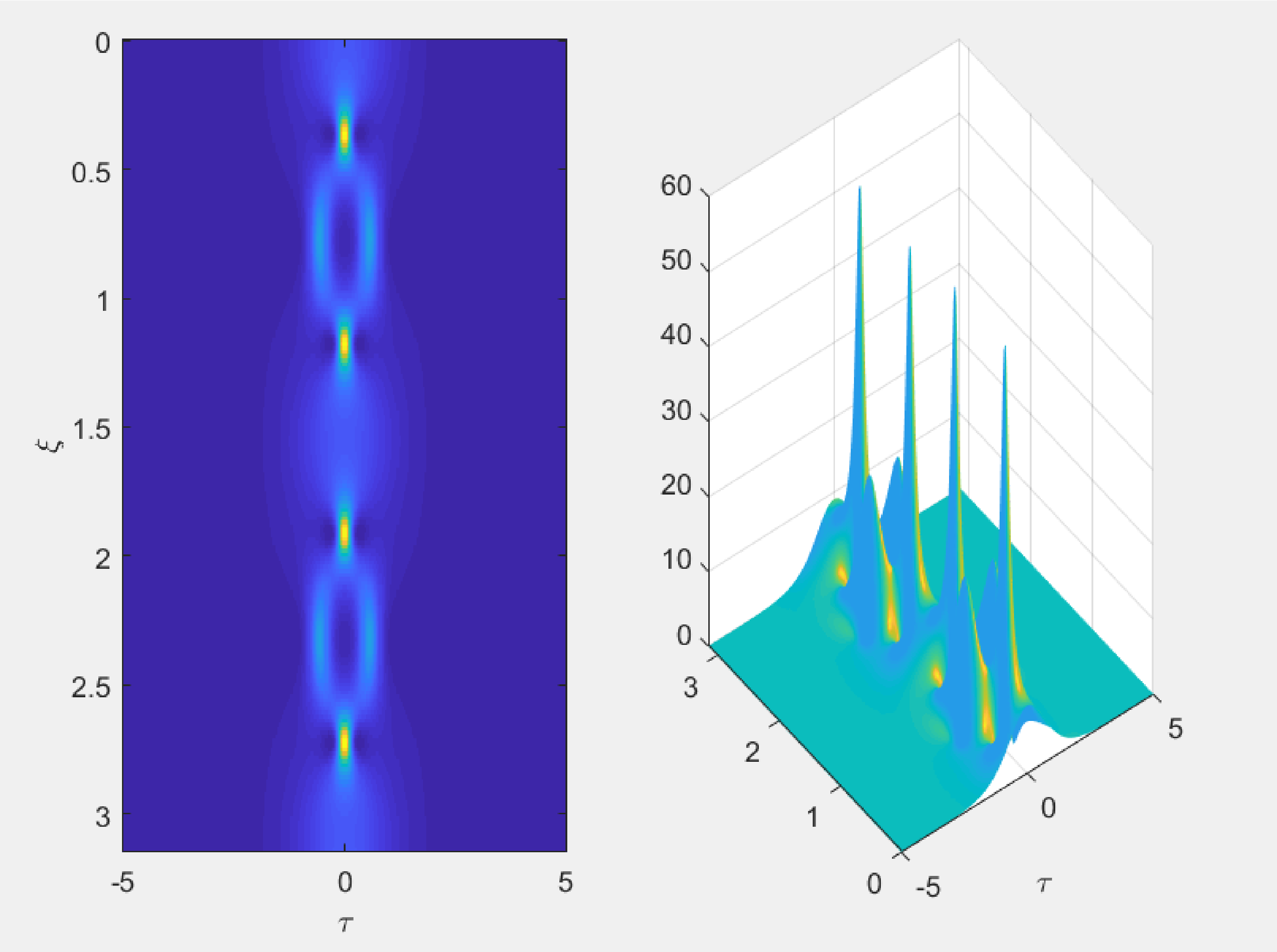


N=2 soliton (higher order soliton)

$$\psi(\tau,0) = N\,\mathrm{sech}(x)$$
$$N = 2$$



N=3 soliton



Analytical solutions (see later)
for N sech as initial condition
tell us that we have
periodical dynamics with period pi/2
for any integer N

Before the «Nature Whatever» era

284 Supplement of the Progress of Theoretical Physics, No. 55, 1974

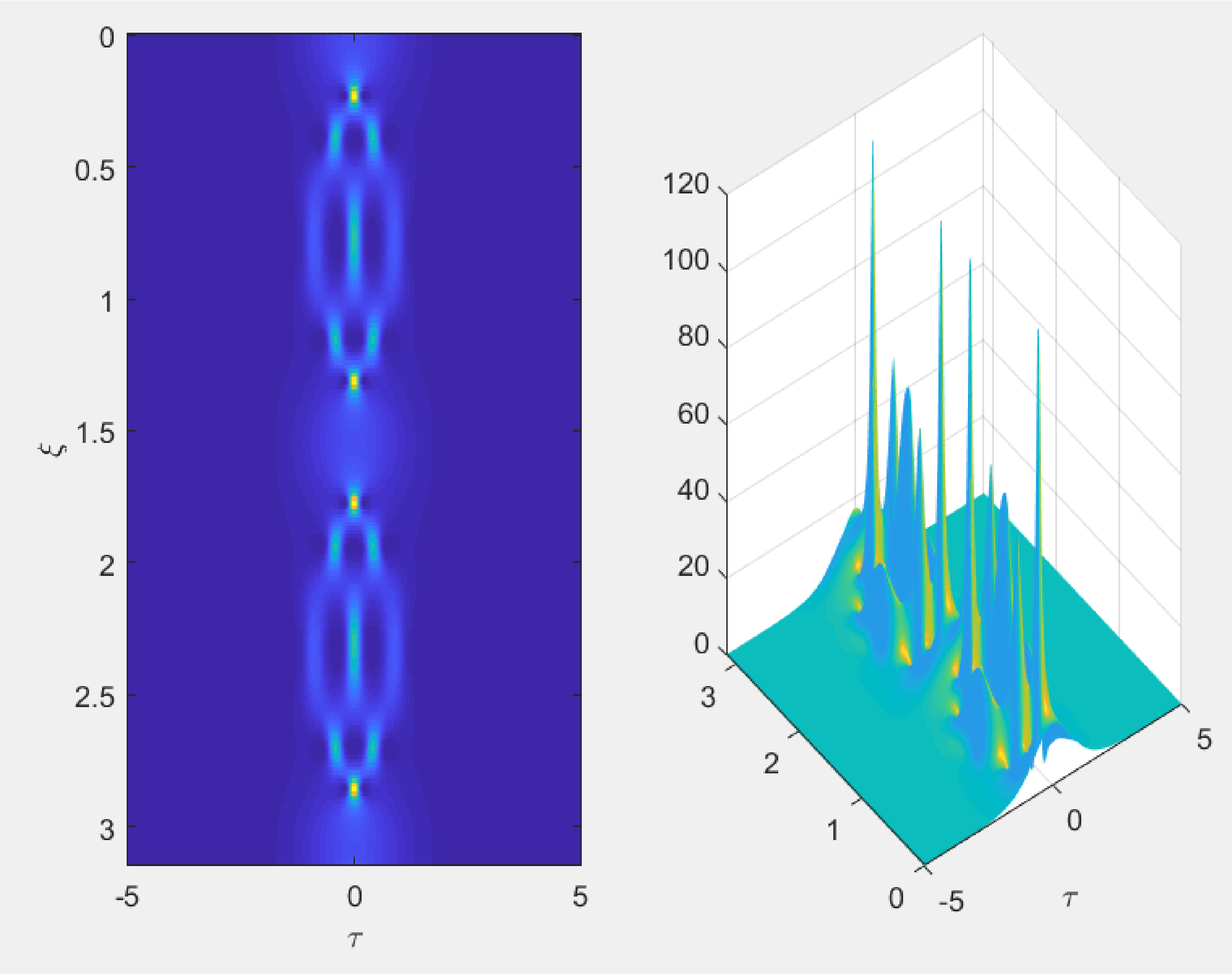
B

Initial Value Problems of One-Dimensional Self-Modulation of Nonlinear Waves in Dispersive Media

Junkichi SATSUMA and Nobuo YAJIMA*

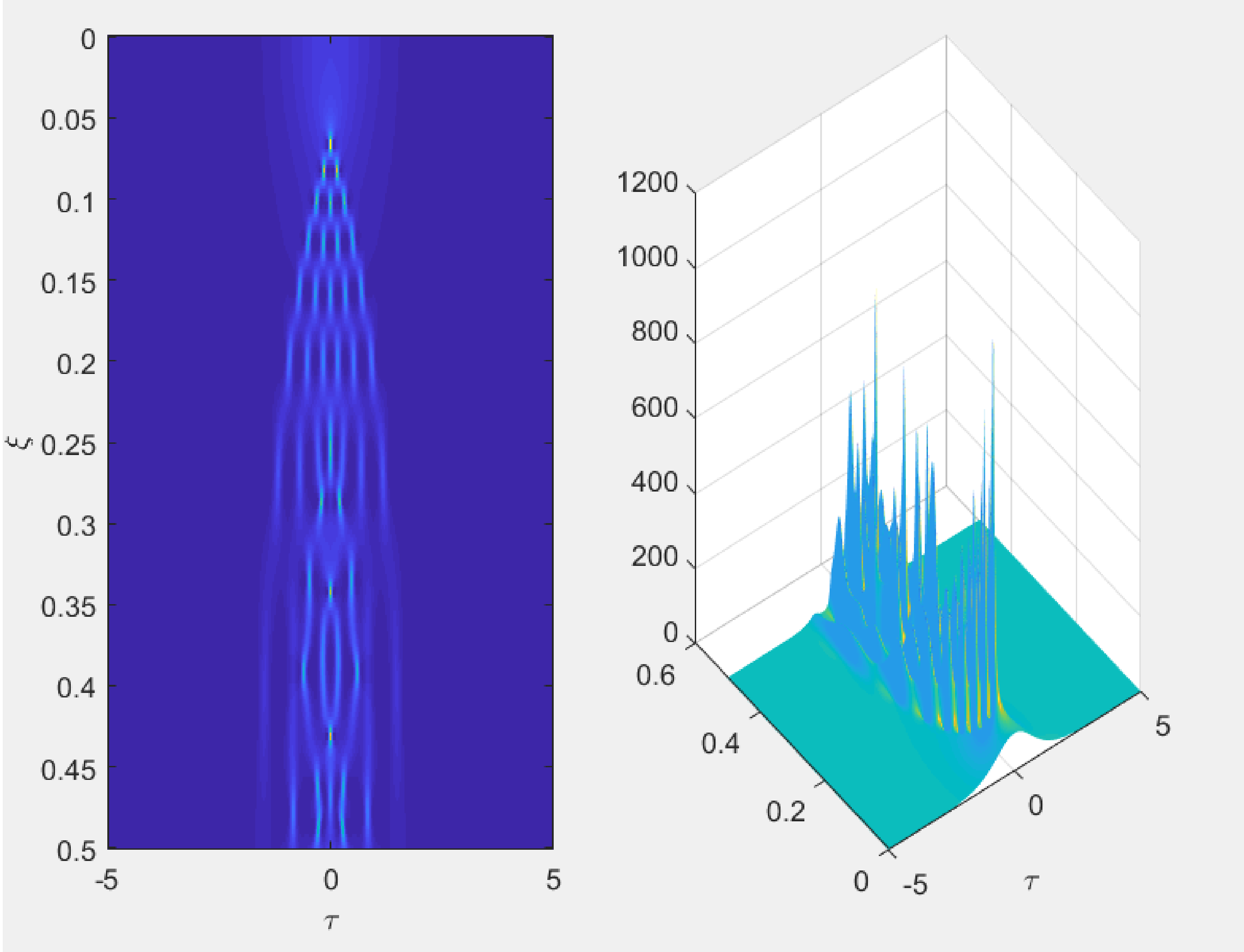
Department of Applied Mathematics and Physics
Kyoto University, Kyoto
**Research Institute for Applied Mechanics*
Kyushu University, Fukuoka

N=4



N=10

PROBLEMS HERE !!!!



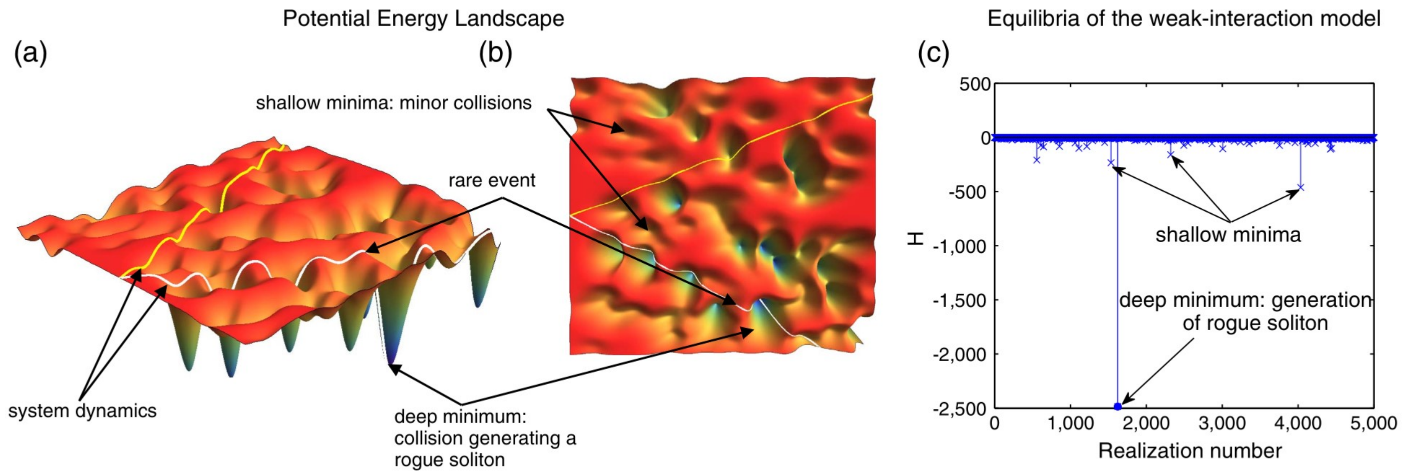
NLP2019

For large N «dynamical complexity» emerges

The system has a «landscape» of states and visits them in a way that is dependent on the history

Numerical noise = temperature

Large sensitivity to any form of noise



PHYSICAL REVIEW E **72**, 066620 (2005)

Complex light: Dynamic phase transitions of a light beam in a nonlinear nonlocal disordered medium

Claudio Conti*
Research Center "Enrico Fermi" Via Panisperna 85/A 00184, Rome, Italy
and Reserch Center SOFT INFM-CNR, University "La Sapienza," P. A. Moro 2, 00185, Rome, Italy
(Received 10 December 2004; revised manuscript received 3 August 2005; published 30 December 2005)

Research Article Vol. 2, No. 5 / May 2015 / *Optica* 497

optica

Rogue solitons in optical fibers: a dynamical process in a complex energy landscape?

ANDREA ARMAROLI,[1,2] CLAUDIO CONTI,[3] AND FABIO BIANCALANA[1,4,*]

Analytical solution of the NLS

Inverse scattering theory

Fourier linear evolution

$$\psi(x,0) = \frac{1}{2\pi}\int \psi(k,0)e^{-ikx}dk$$

$$i\frac{\partial\psi}{\partial z} + \frac{\partial^2\psi}{\partial x^2} = 0 \qquad\qquad i\frac{\partial\psi}{\partial z} - k^2\psi = 0$$

$$\psi(k,z) = \psi(k,0)\exp\left(-ik^2 z\right)$$

$$\psi(x,z) = \int \psi(k,0)\exp\left(-ik^2 z\right)e^{ikx}dk$$

Evolution in the spectral domain (linear case)

Expand	Expand in the initial data in the spectrum (plane waves)
Evolve	Evolve the plane waves
Compose	Compose the evolved plane waves

Evolution in the spectral domain (nonlinear)

Expand	Expand in the initial data in the spectrum (plane waves and solitons)
Evolve	Evolve the plane waves and the solitons
Compose	Compose the evolved plane waves and the solitons

Nonlinear Fourier transform

Fast Numerical Nonlinear Fourier Transforms

Sander Wahls, *Member, IEEE,* and H. Vincent Poor, *Fellow, IEEE*

arXiv:1402.1605

NLP2019

The scattering problem for the nonlinear FT

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2u=0,$$

$$i\frac{\partial v_1}{\partial \tau}+uv_2=\zeta v_1,$$
$$i\frac{\partial v_2}{\partial \tau}+u^*v_1=-\zeta v_2,$$

The nonlinear Fourier transform

As in linear systems, for the NLS we can define a «spectrum»

For the NLS the spectrum is made by the standard continuous spectrum and by a discrete number of solitons

Calculating the spectrum – however – is not as easy as doing a linear Fourier transform

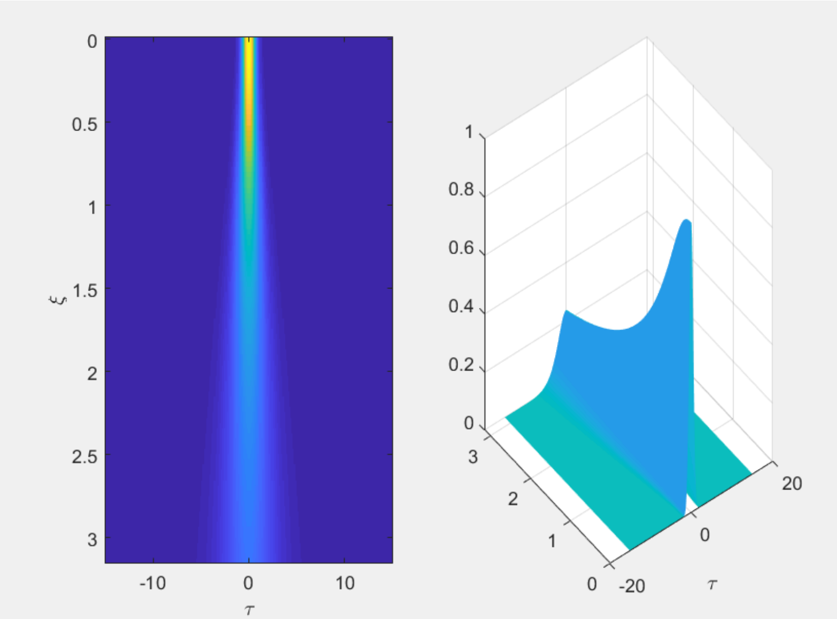
Nonlinear Fourier transform

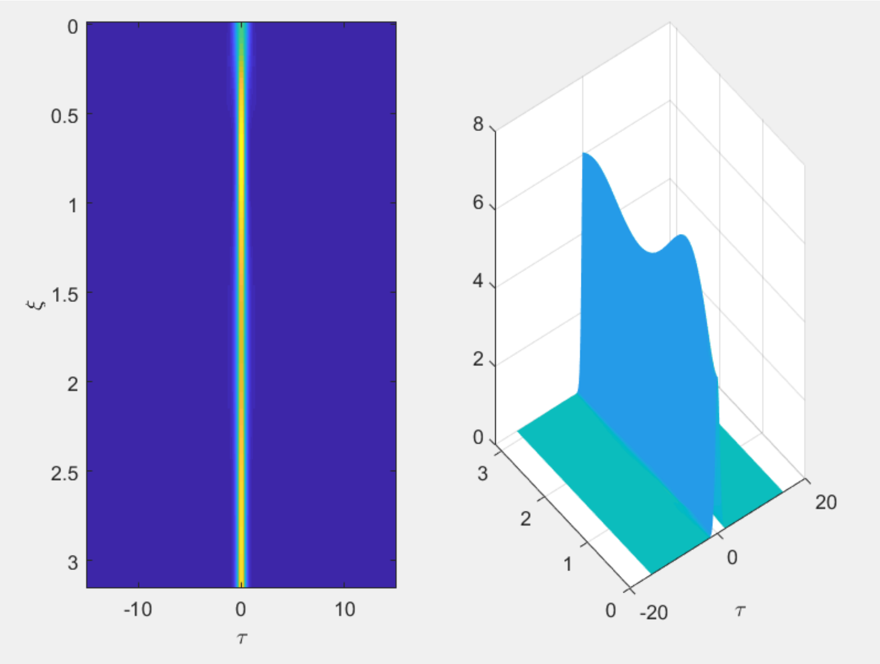
- Continuous spectrum
- Discrete solitons

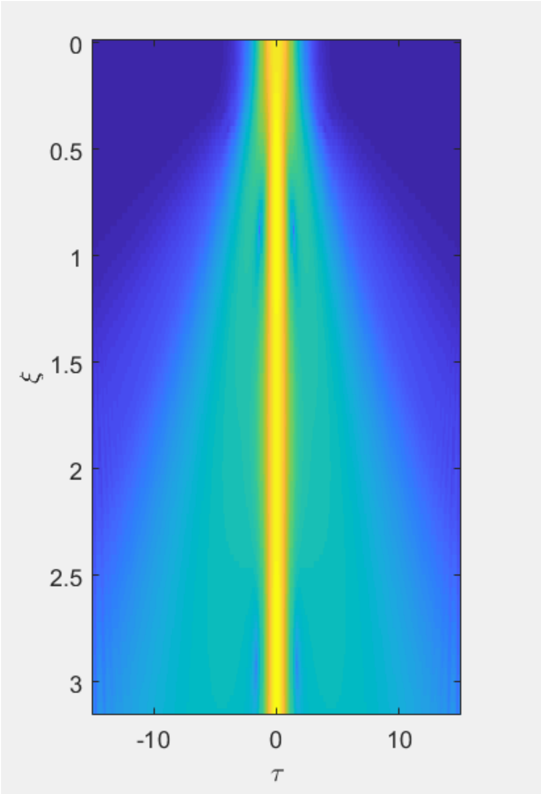
Example by our matlab code

Evolution of a sech (only discrete spectrum, one or more solitons)

Evolution of a Gaussian (discrete and continuos spectrum, varying the input amplitude)







Applications

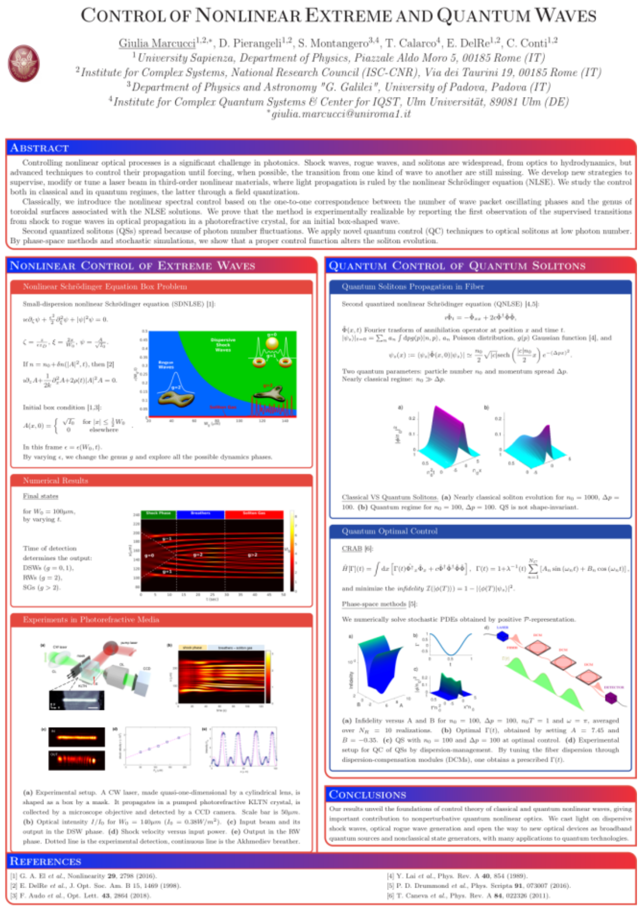
- Nonlinear telecommunications
- Novel quantum sources
- High power lasers
- Ultra-broad band sources (hollow core fibers)

Problem

When the number of solitons grows (hydrodynamic limit) both the numerical methods and the analytical solutions get into trouble

Non trivial phenomena emerge related to rogue waves, shock waves and recurrence

These complex regimes need both advanced numerical and analytical techniques (see poster by Giulia and the lessons by Stefano)



Quantum solitons

The quantum nonlinear Schrödinger equation

Bethe ansatz

Exact solution due to Bethe 1931

$$\imath\partial_t\hat{\phi} = -\hat{\phi}_{xx} + 2c\hat{\phi}^\dagger\hat{\phi}\hat{\phi}$$

The second-quantized Hamiltonian of the NLS is

$$\hat{H} = \int \mathrm{d}x \left(\hat{\phi}_x\hat{\phi}_x^\dagger + c\hat{\phi}^\dagger\hat{\phi}^\dagger\hat{\phi}\hat{\phi}\right), \qquad \imath\partial_t \left|\psi\right\rangle = \hat{H}\left|\psi\right\rangle$$

Superposition of states with n particles

$$|\psi\rangle = \sum_n \frac{a_n}{\sqrt{n!}} \int f(x_1, x_2, \ldots, x_n, t)\hat{\phi}^\dagger(x_1)\hat{\phi}^\dagger(x_2)\ldots\hat{\phi}^\dagger(x_n)\,\mathrm{d}x_1\,\mathrm{d}x_2\ldots\mathrm{d}x_n|0\rangle$$

$$\sum_{n=0}^{\infty}|a_n|^2 = 1 \qquad \int |f_n(\mathbf{x})|^2\,\mathrm{d}\mathbf{x} = 1$$

Exact equation for the distribution function

$$\imath \partial_t f_n(\mathbf{x}, t) = \left[-\sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \le i < j \le n} \delta\left(x_j - x_i\right) \right] f_n\left(\mathbf{x}, t\right)$$

Bethe ansatz (sum over permutations P)

$$f_n = \sum_P A_p \exp\left(\imath \sum_{j=1}^{n} k_{P(j)} x_j \right)$$

$$k_j = p + \imath \frac{c}{2}(n - 2j + 1)$$

$$f_n(\mathbf{x}) = \mathcal{N}_n \exp\left[\imath p \sum_j x_j + \frac{c}{2} \sum_{1 \le 1 < j \le n} |x_i - x_j| \right]$$

n particle eigenstates with momentum p

$$|n,p\rangle = \int \frac{f_{n,p}(\mathbf{x})}{\sqrt{n!}} \phi^\dagger(x_1)...\phi^\dagger(x_n)\,\mathrm{d}\mathbf{x}\,|0\rangle$$

$$|n,p,t\rangle = e^{\imath E_{n,p} t}\,|n,p\rangle$$

$$E_{n,p} = np^2 - \frac{|c|^2}{12} n(n^2-1)$$

Eigenstate of the photon number and momentum operators

$$\hat{N} = \int \mathrm{d}x \hat{\phi}^\dagger(x)\phi(x) \qquad \hat{P} = -\imath \int \mathrm{d}x \hat{\phi}^\dagger(x)\phi_x(x)$$

$$\hat{N}\,|n,p\rangle = n\,|n,p\rangle$$

$$\hat{P}\,|n,p\rangle = np\,|n,p\rangle$$

These states are **not** localized solitons

$$\langle n,p|\,\phi(x)\,|n,p\rangle = 0$$

The quantum soliton state (Lai e Haus 1989)

Soliton state

The field expectation $\hat{\phi}$ is the classical soliton

$$\langle\psi_s|\,\hat{\phi}\,|\psi_s\rangle = \psi_s(x,t)$$

$\psi_s(x,t)$ solution of classical nonlinear Schrödinger equation.

Time-dependent solution of the **linear** quantum Schrödinger

$$|\psi_s\rangle = \sum_n a_n \int g_n(p)\,|n,p,t\rangle\ \mathrm{d}p$$

$$\sum_n |a_n|^2 = 1 \qquad\qquad \int |g_n(p)|^2\,\mathrm{d}p = 1$$

The quantum soliton state (simple)

$$a_n = \frac{\alpha_0^n}{\sqrt{n!}} e^{-\frac{|\alpha_0|^2}{2}} \qquad g_n(p) = \frac{1}{\sqrt{\sqrt{\pi}\Delta p}} e^{-\frac{|p-p_0|^2}{2\Delta p^2}}$$

Two parameters:

- α_0 gives number of bosons n_0 and phase, $|\alpha_0|^2 = n_0$
- Δp momentum spread

At $t = 0$ we have

$$\langle\psi_s|\hat{\phi}|\psi_s\rangle = \sum_n \frac{|\alpha_0|^{2n}}{n!} \frac{\alpha_0\sqrt{n(n+1)}}{2} |c|^{1/2} sech\left[\frac{1}{2}\left(n+\frac{1}{2}\right)|c|x\right] e^{-\Delta p^2 x^2}$$

Classical limit

When $n_0 = |\alpha_0|^2 >> 1$ and for $\Delta p \to 0$ we have

$$\langle\psi_s|\,\hat{\phi}\,|\psi_s\rangle \cong \psi_s(x,t) = n_0\sqrt{2|c|} sech\left(|c|^{1/2} n_0 x\right)$$

Position and momentum fluctations

Position operator

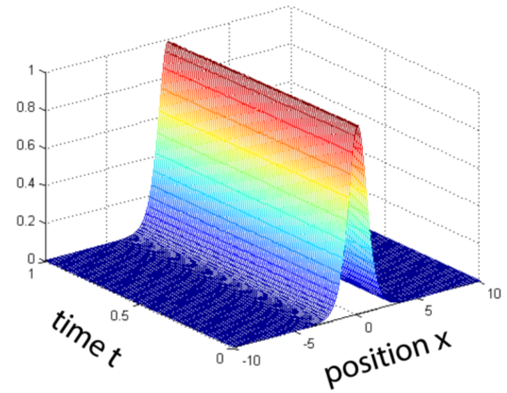
$$\hat{X} = \left[\int x\hat{\phi}^\dagger(x)\phi(x)\,\mathrm{d}x\right]\hat{N}^{-1} \text{ with } [\hat{X},\hat{P}] = \imath$$

One finds

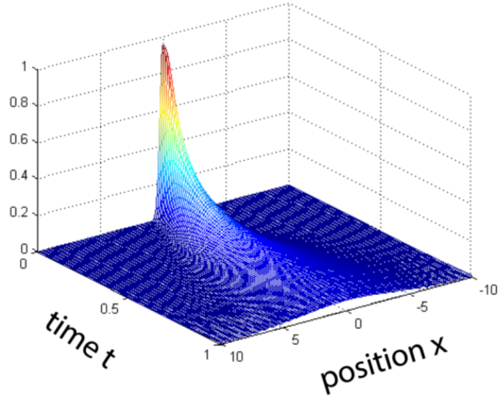
$$\langle \Delta P^2\rangle \cong (n_0 \Delta p)^2$$

$$\langle \Delta X^2\rangle \cong \frac{1}{4\Delta p^2 n_0^2} + 4\Delta p^2 t^2$$

Classical soliton



Quantum $n_0 = 50$, $\Delta p = 100$

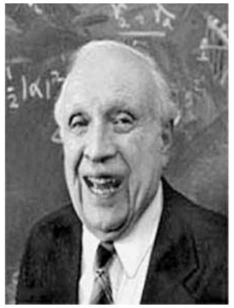


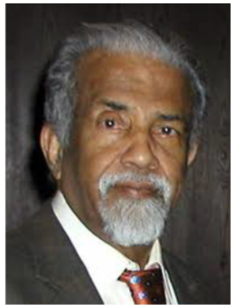
Simulate the Quantum NLS?

We want to numerically validate the quantum spreading.
Phase-space approach: the Positive P-rapresentation

Positive Glauber-Sudarshan P-rapresentation

Map a nonlinear field theory to c-number stochastic equations (Sudarshan 1963; Glauber 1963,Drummond and Gardiner 1980)





- One expands the density matrix ρ in two sets of coherent states spanned by complex parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

$$\rho = \int P(\boldsymbol{\alpha}, \boldsymbol{\beta}) \frac{|\boldsymbol{\alpha}\rangle \langle \boldsymbol{\beta}^*|}{\langle \boldsymbol{\beta}^* | \boldsymbol{\alpha}\rangle} \mathrm{d}\mu(\boldsymbol{\alpha}, \boldsymbol{\beta})$$

- A Fokker-Planck equation for probability distribution
- An equivalent Itô stochastic differential equations for c-numbers; nonlinearity introduce noise terms.

Example of use of the Positive P-rapresentation

- Harmonic Oscillator $\hat{H} = \hbar\omega a^{\dagger}a$ equivalent to the stochastic equation

$$\frac{d\alpha}{dt} = -\imath\alpha$$

- Nonlinear Harmonic Oscillator $\hat{H}_{int} = \frac{\hbar\kappa}{2}{a^{\dagger}}^2 a^2$ is equivalent to the coupled stochastic (notice: two c-numbers for any ladded operator)

$$\frac{d\alpha}{dt} = -\kappa\alpha^2\beta + \imath\sqrt{\kappa}\alpha\xi_1(t)$$

$$\frac{d\beta}{dt} = -\kappa^*\beta^2\alpha - \imath\sqrt{\kappa^*}\beta\xi_2(t)$$

NLP2019

Stochastic Partial Differential Equations

The Fokker-Planck equation is equivalent to two coupled fields Ito SDE ϕ and ψ. The quantum NLS Hamiltonian

$$\hat{H} = \int \mathrm{d}x \left(\hat{\phi}_x \hat{\phi}_x^\dagger + c \hat{\phi}^\dagger \hat{\phi}^\dagger \hat{\phi} \hat{\phi} \right)$$

One has the corresponding *Stochastic Differential Equations*

$$\partial_t \phi = -\imath \partial_x^2 \phi - \imath 2 c \phi^2 \psi + \sqrt{\imath c} \xi_\phi(t,x) \phi$$

$$\partial_t \psi = \imath \partial_x^2 \psi + \imath 2 c \phi \psi^2 + \sqrt{-\imath c} \xi_\psi(t,x) \psi$$

Classical limit

$\xi \to 0$ (no quantum noise), and $\psi = \phi^*$ and one obtains the classical NLS

$$\imath \partial_t \psi = -\psi_{xx} + 2c|\psi|^2 \psi$$

Stochastic Runge Kutta Pseudospectral Algorithm

We solve numerically the stochastic nonlinear partial differential equation

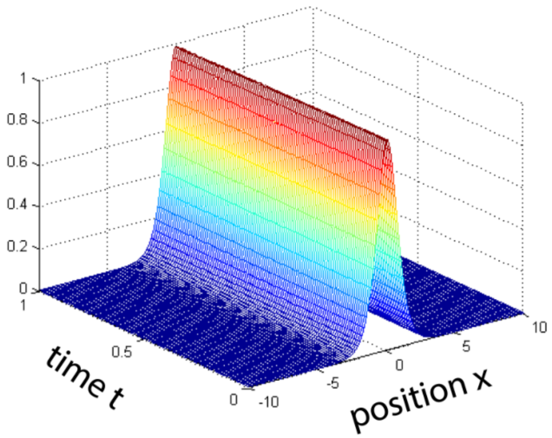
- We discretize the spatial variable x
- We adopt a pseudospectral approach for derivatives (PDE$\rightarrow$ODE)
- We adopt a second-order stochastic Runge Kutta algorithm ($u = (\phi, \psi)$) with the Itô rule $\mathrm{d}W^2 = \mathrm{d}t$

$$
\begin{aligned}
u_{k+1} &= u_k + \tfrac{F_1}{2} + \tfrac{F_2}{2} \\
F_1 &= \mathrm{d}t D(u_k) + S(u_k)\,\mathrm{d}W_1 \\
F_2 &= \mathrm{d}t D(u_k + \mathrm{d}t F_1) + S(u_k + \mathrm{d}t F_1)\,\mathrm{d}W_2
\end{aligned}
$$

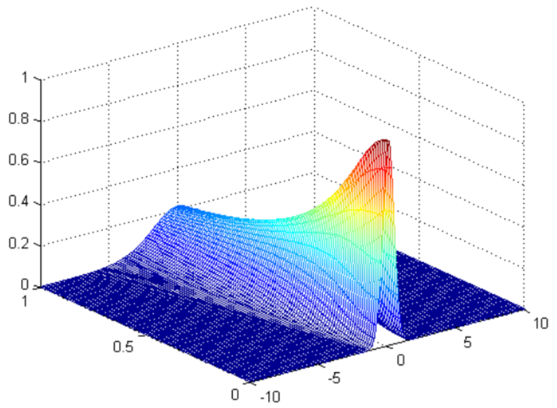
where $D(u) = -\phi_{xx} + c\phi^2\psi \ldots$ is the deterministic part, and $S = \sqrt{ic}\phi$ following the coherent state expansion.

Simulation of the quantum soliton (1/2)

- Initial condition for the exact quantum soliton (Lai and Haus theory)
- Average over disorder realization
- We have two parameters:
  1. n_0 determining the number of photons, which fixes the strength of quantum noise
  2. Δp the momentum spread of the photon states (here $\Delta p = 10$)



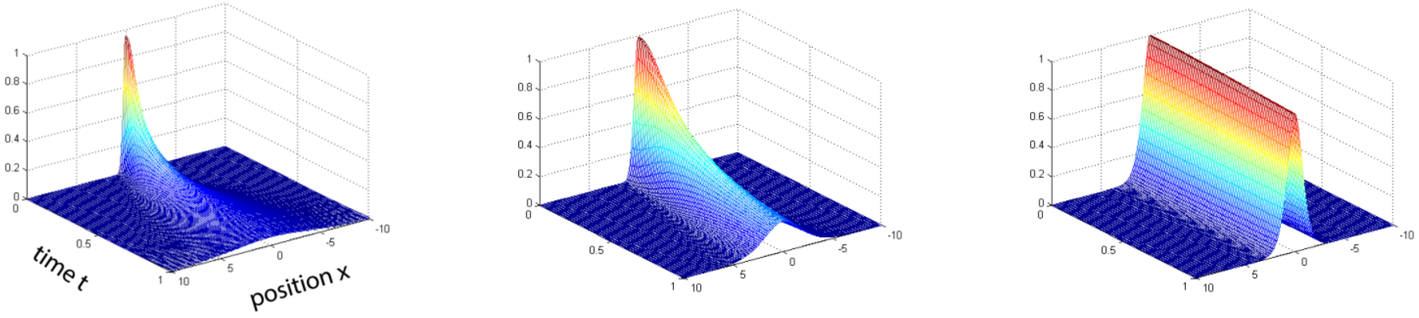
Classical-like evolution $n_0 = 1000$



Quantum evolution $n_0 = 10$

Simulation of the quantum soliton (2/2)

For a fixed momentum spread, we can change the number of photons to transit from classical to quantum ($\Delta p = 100$).



$n_0 = 50$ $n_0 = 100$ $n_0 = 1000$

Low-particle number solitons

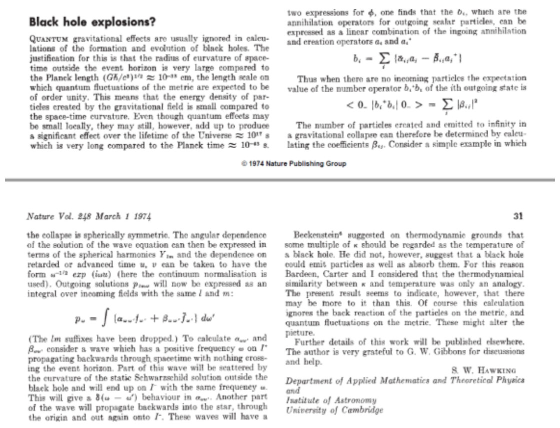
Low-particle number solitons exists but are delocalized!

Do quantum soliton evaporate ?

Hawking radiation from black holes

Nature, 1974

$$\phi_{;ab}\, g^{ab} = 0$$



Black hole explosions?

QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes. The justification for this is that the radius of curvature of space-time outside the event horizon is very large compared to the Planck length ...

Nature Vol. 248 March 1 1974

S. W. HAWKING

© 1974 Nature Publishing Group

The spectrum of a quantized field in the black hole metrics (Schwarzschild solution) is blackbody with temperature

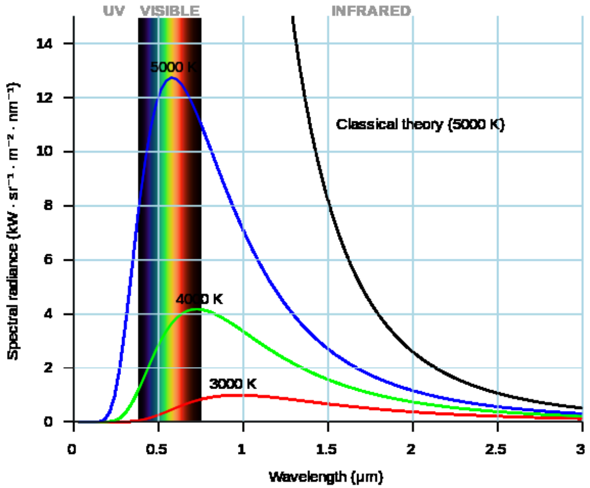
$$T = \frac{\hbar c^3}{8\pi G M k_B} \quad \left(\approx \frac{1.227\times 10^{23}\ \mathrm{kg}}{M}\ \mathrm{K} = 6.169\times 10^{-8}\ \mathrm{K}\times\frac{M_\odot}{M}\right)$$

$$\nu_{\max} = T \times 58.8\ \mathrm{GHz\ K^{-1}}$$

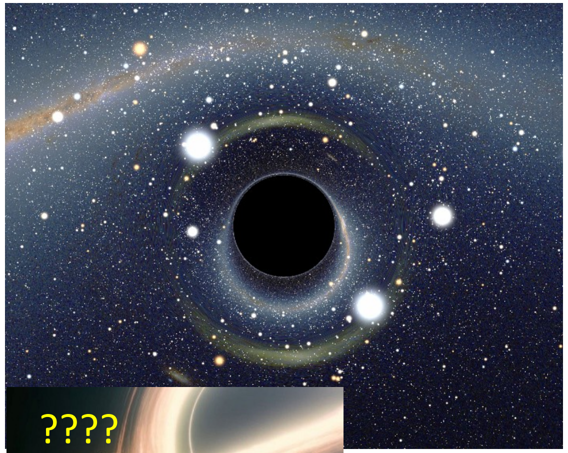
$$\lambda_{max} = \frac{b}{T_H} = \frac{8\pi^2}{4.9651} r_s = 15.902\, r_s$$

Black hole classifications

Class	Mass	Size
Supermassive black hole	~10^5–10^{10} M_{Sun}	~0.001–400 AU
Intermediate-mass black hole	~10^3 M_{Sun}	~10^3 km ≈ R_{Earth}
Stellar black hole	~10 M_{Sun}	~30 km
Micro black hole	up to ~M_{Moon}	up to ~0.1 mm



Wikipedia



«Interstellar» movie

Black holes are solitons

Volume 61B, number 4 PHYSICS LETTERS 12 April 1976

BLACK HOLES AS SOLITONS

A. SALAM
International Centre for Theoretical Physics, Trieste, Italy, and Imperial College, London, England

and

J. STRATHDEE
International Centre for Theoretical Physics, Trieste, Italy

Received 2 February 1976

We remark that exact classical Schwarzschild-like solutions to Einstein's (and possibly f gravity) equations provide examples of realistic solitons.

Under the broadest definition, any non-trivial solution to a system of classical non-linear equations, which is confined to a finite region of space and which carries a finite energy, may be considered a soliton. The problem is to discover to what extent such classical objects can approximate to the quantum systems encountered in particle physics. Are they stable? What conserved quantities can be associated with them? How do they interact with "ordinary" particles described by quantized fields?

Black holes are solitons of the Einstein-Hilbert equations …

Black holes evaporate ….

Do all kinds of solitons evaporates?

Temperature of an optical soliton ?

Quantum soliton evaporation

IOP Publishing J. Phys. Commun. 2 (2018) 055016 https://doi.org/10.1088/2399-6528/aac340

Journal of Physics Communications

PAPER

CrossMark

OPEN ACCESS

RECEIVED
9 April 2018

Sine-Gordon soliton as a model for Hawking radiation of moving black holes and quantum soliton evaporation

Leone Di Mauro Villari[1,2], Giulia Marcucci[2,3], Maria Chiara Braidotti[2,4] and Claudio Conti[2,3]

PHYSICAL REVIEW A **98**, 043859 (2018)

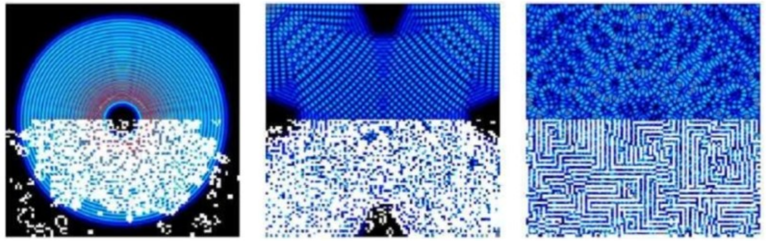
Quantum soliton evaporation

Leone Di Mauro Villari,[1,2] Daniele Faccio,[1,3] Fabio Biancalana,[1] and Claudio Conti[2,4]

[1]*Institute of Photonics and Quantum Sciences, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom*
[2]*Institute for Complex Systems, National Research Council (ISC-CNR), Via dei Taurini 19, 00185 Rome, Italy*
[3]*School of Physics and Astronomy, Kelvin Building, University of Glasgow, Glasgow G12 8QQ, United Kingdom*
[4]*Department of Physics, University Sapienza, Piazzale Aldo Moro 5, 00185 Rome, Italy*

More complex,
nonlinear and quantum curiosity?

- Talks by Stefano and Arno Mussot
- Poster by Giulia



www.newcomplexlight.org